OUTLINES

OF

APPLIED OPTICS

NUTTING

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OUTLINES

OF

APPLIED OPTICS

BY

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PREFACE.

These outlines of applied optics deal with optical instruments and optical measurements from the standpoint of sensibility and precision. The first three chapters treat of instruments for forming images; the remaining chapters, special instruments for analyzing light and determining the properties of materials. The keynote throughout is the question of securing the best possible results in optical work. It might be well classed as optical engineering or technical optics, but applied optics is a broader term.

Applied optics is practically untaught in any university. By the student of pure optics, optical instruments are regarded as mere tools to be simplified and ignored rather than studied. The physical properties of the eye and photographic plate, essential parts of every optical instrument, are largely unknown and disregarded. Color and colorimetry rest practically where Maxwell's great contributions left them. Light itself is not even precisely defined.

No class of engineering offers higher prizes than the different branches of optical engineering-lens design, illuminating engineering, colorimetry, photography, radiometry, pyrometry, etc. No richer field awaits the investigator versed in pure optics than those of applied optics, and a student can find no more alluring, promising, or brain racking problems than are to be found in these neglected fields. There are ample reasons why applied optics should be taught as such in at least a few of our leading universities, and it is hoped that this work may add strength and unity to such tendencies.

But the book has been prepared for the worker in applied optics rather than the student; for the men in the field

designing instruments, measuring color, examining eyes, identifying illuminants, etc., who may find a suggestion of how to obtain better results or ready information on nearly related subjects.

A full treatment of applied optics of the scope here chosen could be adequately treated only in a number of volumes by a dozen specialists, but as the time is not yet ripe for so extended a treatise, it was thought best to prepare a briefer work of the same scope to serve as an entering wedge. More than all else, it is hoped that this book will stimulate work in the many almost unworked fields within its borders so that when the time is ripe, the material for a more pretentious treatise may be available.

The material for this work was obtained almost entirely from the original papers to which references are made. Descriptions of instruments described in text-books are omitted. In the first chapter the well known works of Dennis Taylor and of Whittaker are freely drawn upon. I am indebted to Mr. E. D. Tillyer, Dr. F. E. Wright, Dr. W. W. Coblentz and other colleagues for careful reading of parts of the manuscript.

P. G. Nutting.

Washington, D. C.

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OUTLINES OF APPLIED OPTICS.

INTRODUCTION.

Pure and Applied Optics.—Optics falls naturally in three divisions; pure optics, applied optics and cosmical optics. *Pure optics* treats of the laws and phenomena connected with the emission, propagation and absorption of light, from either the physical or mathematical standpoint. *Applied optics* deals with the applications of optical principles to optical observations and measurements, hence with the design, use and properties of all kinds of optical instruments and their accessories. Subdivisions of applied optics deal chiefly with the *direction* of the light, its *intensity*, *quality* and state of *polarization.* Photometry, spectrophotometry, colorimetry, optometry, photography, optical pyrometry, polarimetry, interferometry and many other subjects are included in applied optics. *Cosmical optics* deals with terrestrial, planetary, solar and stellar optical phenomena. This introduction to applied optics is a summary of those parts of pure optics of most interest to the technologist.

Light.—Psychologically light is a sensation, physiologically, radiation capable of stimulating the visual organs and causing the sensation. The known spectrum has in recent years been extended from the visible far into the ultra-violet (0.1μ) by photography and into the infra-red (to 100μ) by radiometry and since no definite limits to the visible spectrum can be set, light is here used in a figurative sense, to include the radiation of the entire known spectrum (0.1 to 100μ),

but not electric waves (4 mm to 10 km), Röntgen rays or similar related radiation.

Light is supposed to consist of pulses (continuous spectra) or of short trains (discontinuous spectra) of waves, the pulses or trains numerous and much jumbled up under ordinary conditions, but each travelling its own straight line path independently of the others. In each train the waves run about 50,000 to the inch or 2000 to the millimeter (blue-green light), such waves are 0.0005 mm $=0.5\mu$ $=500\mu\mu$ in length. In ordinary (white light) vision, we use chiefly waves between 0.500 and 0.640μ in length, although radiation of wave lengths 0.400 and 0.800μ is easily visible or even 0.32 and 1.0μ if sufficiently intense. Ordinary photographic plates are sensitive to the blue-green, the violet and ultra-violet. Plates may be sensitized to the yellow and red, but not to the infra-red.

The Spectrum.—In the ultra-violet, glass transmits freely out to 0.30–0.33μ, calcite to about 0.23μ, quartz, water and air to 0.18μ, fluorite to at least 0.12μ. The spectrum of

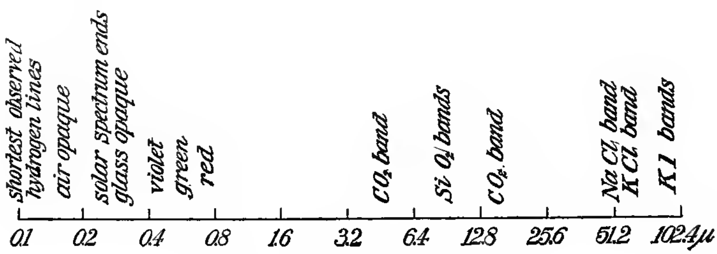

FIG. 1.—The optical spectrum in octaves.

hydrogen has been studied to 0.11μ, the present limit of exploration. The solar spectrum ends quite abruptly at 0.293μ. Conducting gases in general (arc, spark and vacuum tube) emit many visible and ultra-violet lines, but very little in the infra-red. At the red end, the spectrum has been extended out to 96.7μ, the selective reflection from potas-

sium iodide. Heated carbon dioxide emits bands at 4.4 and 14.0μ. Water absorbs bands at 3 and at 6μ, quartz and many other silicates selectively absorb and reflect bands at 8.5, 10.0, and 14.0μ. Rock salt reflects a band at 51.2μ, sylvin one at 61.6 and potassium bromide a band at 67μ.

Fig. 1 is a graphical representation of the known spectrum plotted in octaves. There are ten octaves in all, two in the ultra-violet, one visible and seven in the infra-red. Electric waves, supposed similar to light waves but longer, have been produced and studied as short as 4 mm, five octaves beyond the last point on the scale in the figure.

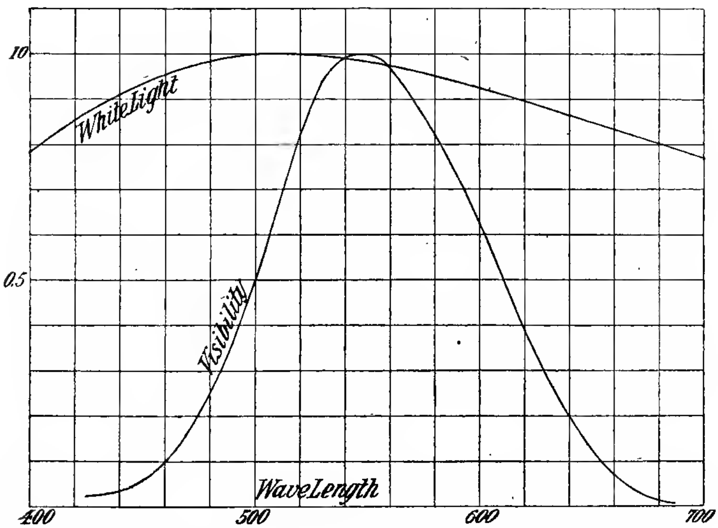

FIG. 2.—Visibility of radiation and spectral energy of white light.

Visibility and Luminosity.—At wave length 500 in the blue-green and at 610 in the red, the eye is only half as sensitive as in the yellow-green at 545. That is, to produce the same sensation of brightness, radiation must be twice as intense in the blue-green and red as in the yellow-green.

Similarly in the deep blue at 460 and deep red at 660, the visibility of radiation is but a tenth that at 545. The complete visibility curve is given in Fig. 2.

This curve represents the relative sensibility of the eye to radiation of the different wave lengths. It has not yet been accurately determined, but its height at each wave length is not very different from that given below:

Wave length	460	480	500	520	540	560	580	600	620	640	660
Visibility	.12	.26	.47	.82	.99	.95	.79	.60	.41	.22	.08

If the distribution of energy in the spectrum of any source of light be known, the corresponding distribution of light in its spectrum may be found by multiplying the energy (watts per sq cm per unit wave length) by the visibility at each point. The product gives the luminosity curve of the source and the integral of the luminosity is the total light emitted by the source. In Fig. 2 are plotted the energy curve and the corresponding luminosity curve for sunlight at the earth's surface (zenith sun, Washington).

Emission.

Origin of Light.—Nearly all light originates either in bodies heated to incandescence (sun, glow lamp, common arc, gas flame) or in gases conducting an electric current (flaming arc, mercury tube, spark). Since light is supposed to consist of short electromagnetic pulses and waves, and such waves can originate only in the accelerated motions of electrical charges, the origin of light is thought to be in probably all cases either the rotations and vibrations (line spectra, wave trains) or the stopping and starting (continuous spectra, pulses) of small charges (electrons) forming part of every atom. The electric current or mere heating would violently agitate these particles and thus cause the emission of pulses or trains of waves. Since little is known of the character of these motions, the quantitative theory of

emission is yet undeveloped and the few known emission laws are empirical.

Sources of Light.—Light sources differ from one another in regard to (1) intensity and (2) quality of the light emitted. By passing the light through a grating or prism in the proper manner it may be resolved into parts of different wave lengths called its spectrum, each part being of the same wave length. It is found that with a single exception, every light source has its own characteristic spectrum not exactly like any other. All bodies heated in a hollow enclosure with opaque walls (perforated at one point for viewing), give out the same spectrum as the interior walls. This spectrum is independent of the nature of the walls, but varies with the temperature of the walls. Every source is then characterized by its spectrum, by the energy of its radiation of each wave length; this depends in turn upon the nature of the source and the intensity of its excitation.

Spectra are of two general classes, continuous spectra and line spectra. *Continuous spectra* are emitted by incandescent solids and dense vapors. Bodies emit reddish light (if any) when heated to 500 to 1000°, sensibly white light at 2000° and over. The crater of an electric arc is at a temperature of 3700 to 4000°. The most luminous layer of the sun is at a temperature of about 6000° C. Continuous spectral energy curves differ locally from each other (see Chapter IX), but all are supposed to lie under that of a full radiator, the interior of a heated opaque envelope. Sunlight is of course our standard of white light, being the light to which our eyes are most accustomed. The nearest white of artificial sources is probably the carbon dioxide tube, then come the acetylene flame, the Nernst lamp, tungsten lamp, and Welsbach mantle. All of these sources (including sunlight) fluctuate in quality to such an extent that a standard of white light is very difficult to define or reproduce.

Line spectra are usually obtained from the arc, spark, or vacuum tube, which are all merely arrangements for passing an electric current through a gas. Gases and vapors emit

line spectra if merely heated to a high temperature (over 3000° C) in an electric oven. At lower temperatures (500 to 1000°) they emit no visible radiation that can be detected, but some compound vapors, notably water vapor and carbon dioxide, emit characteristic bands in the infra-red, as do many crystalline salts. The Bunsen and oxyhydrogen flames give the line spectra of introduced salts or metals if these combine or decompose at their temperatures.

Arc, spark and vacuum tube spectra are rich in visible and ultra-violet lines. The *arc* gives the spectrum of the electrodes or of any salt or bead of metal introduced into it, showing little or none of the spectrum of the air or other ambient gas. It is easy to manipulate, but difficult to keep constant. The *spark* gives not only the spectra of the electrodes or introduced substances, but of the ambient gas. The spark spectra of most substances contain many lines additional to those of the arc. The *vacuum* (or Plücker) *tube* is best for permanent gases and substances easily vaporized. Both sparks and tubes are best operated with a 5000 or 10,000 volt transformer controlled by resistance in the primary, but are often operated on an induction coil.

In visual observation there are two convenient reference points by which wave lengths may be estimated by color. These are the neutral point between blue and green, wave length 500, and the mid orange, wave length 600. With a little practice wave lengths 400 in the violet and 700 in the deep red may be estimated and other wave lengths interpolated.

Useful Lines.—In the spectrum of sunlight, even a small hand spectroscope reveals many dark lines. The more prominent of these are called *Fraunhofer lines* and are designated by letters. The wave lengths of these lines are given in the following table:

The Fraunhofer Lines.

Line.	Color.	Wave length.	Source.
A	Red......	759.4 (Band).......	Oxygen in atmosphere.
a	Red......	718.5 (Band).......	Water vapor in atmosphere.
B	Red......	686.7..............	Oxygen in amosphere.
C	Red......	656.3..............	Hydrogen, sun.
D_1D_2	Yellow....	589.6, 589.0........	Sodium, sun.
E	Green.....	527.0..............	Calcium, sun.
$b_1b_2b_4$	Green.....	518.4, 517.3, 516.8..	Magnesium, sun.
F	Blue......	486.1..............	Hydrogen, sun.
G	Violet....	430.8..............	Calcium, sun.
H,K	Violet....	396.9, 393.4........	Calcium, sun.

A plurality of the numerous finer dark lines in the solar spectrum (there are at least 20,000 in all) are due to iron, while nickel, cobalt, manganese, titanium and other metals are well represented.

Convenient Bunsen flame lines are—potassium red 769.9, 766.5 (double); lithium red 670.8; sodium yellow 589.6, 589.0 (double); thallium green 535.1; magnesium green 518.4; strontium blue 460.7. Plücker tubes containing mercury, hydrogen or helium give intense isolated lines convenient as sources. Mercury gives a pair of yellow lines 579.0 and 576.9, a brilliant green line 546.1, blue 491.6 and 435.8, and violet 407.8 and 404.7. Hydrogen gives red 656.3, blue 486.1 and 434.1. Helium gives red 728.2, 706.5 and 667.8, yellow 587.6, three green 504.8, 501.6 and 492.2; blue 471.3 and 447.2; and violet 438.8, 402.6 and 388.8. Copper, zinc cadmium and aluminum (singly or alloyed) give useful spark lines, particularly in the ultra violet, the wave lengths of which may be obtained from any wave length tables. To obtain a continuous background in the ultra violet (for absorption work), a heavy cadmium spark is used. Iron and titanium arcs are most frequently used as reference spectra when numerous lines are desired. The uranium arc

gives the spectrum richest in lines (at least 5000) giving with low resolving power practically a continuous spectrum.

The lines for which the indices of optical glass are usually given are:

Line.	Color.	Wave length.	Source.
A′	Red......	768.2 (mean)........	Potassium flame.
C	Red......	656.3...............	Hydrogen tube.
D	Yellow....	589.3 (mean).......	Sodium flame.
F	Blue......	486.1...............	Hydrogen tube.
G′	Violet....	434.1...............	Hydrogen tube.
H′	Violet....	404.7 (extreme).....	Mercury tube.

Standard Lines.—The wave lengths of three cadmium lines have been determined directly in terms of the standard meter at Paris. They are red 643.84696, green 508.58219 and blue 479.99087$\mu\mu$, corrected to dry air at 15° C and 760 mm pressure. From these have been obtained the wave lengths of fifty or more lines, chiefly in the visible iron spectrum, for use as reference standards of wave length. Work on this table of reference lines is not yet complete. It supersedes Rowland's table of standard wave lengths that has been in use for twenty years.

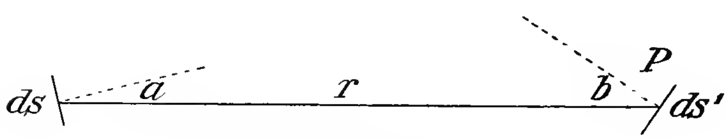

FIG. 3.—Geometry of radiation.

Space Relations.—Several useful space relations follow from Lambert's law and the law of inverse squares. *Lambert's law* states that the radiation per unit area from a surface is proportional to the cosine of the angle (measured from the normal) of emission. A heated ball appears uni-

formly bright all over. A red hot metal plate is of the same brightness viewed at any angle since the foreshortening of the area just compensates for the variation in the radiation from a given area. Lambert's law holds for mat surfaces for both emitted and reflected radiation.

At any point P in space at a distance r from a small element ds of any radiating surface s, and in a direction at an angle a from the normal to ds, it follows from Lambert' slaw that the intensity of the radiation will be

$$dI = I_o \, ds \frac{\cos a}{r^2},$$

where I_o is the intensity at unit distance out on the normal. The total radiation at P (entering a hole of unit area say), from the whole surface of which ds is an element will be the integral of dI over the whole surface.

In the actual measurement of radiation (except with the absolute pyrheliometer) a flat surface is placed at P to receive the radiation and the cosine law must be applied to this surface. In this case

$$dI = I_o \frac{\cos a \cos b}{r^2}.$$

where b is the angle between the ray and the normal to the receiving surface. This expression has been integrated for several practical cases such as cylindrical rod (Nernst filament), circular and rectangular plates and narrow ribbons as sources (see Photometry and Radiometry).

Emission Laws.—The amount of radiation emitted by any body depends upon the wave length, the nature and condition of its surface, its temperature or internal energy, and the refractive index of the surrounding medium, but these relations are known only for the full radiator, the interior of a heated opaque envelope. For such a body *Planck's law*.

$$E_\lambda = C_1 \lambda^{-5} (e^{\frac{C_2}{\lambda T}} - 1)^{-1}$$

(E = emission, λ = wave length, T = temperature, C_1 and

C_2 constants) has been found to hold. For the short waves of the visible spectrum the older *Wien-Paschen law*

$$E_\lambda = C_1 \lambda^{-5} e^{-\frac{C_2}{\lambda T}}$$

is sufficient and more convenient. This function decreases each way from a central maximum, the wave length λ_m of this maximum varies inversely as the absolute temperature T or $\lambda_m T$ = constant. The integral of this function, $E = CT^4$ is *Stefan's law* giving the relation between the total radiation (per unit area) and the absolute temperature T. The value of the constant C is about $10-^{12}$ watt.

Other things being equal, the radiation from any body varies as the square of the *refractive index* of the surrounding medium.

The radiation from surfaces of heated carbon, platinum and other solids and from a few conducting gases have been investigated experimentally (see Chapters X and XI), but the results have not yet been formulated theoretically.

Absorption.

Every known substance absorbs to some extent radiation of some wave length and nearly all absorb very strongly at some particular wave lengths or spectral region. Hydrogen and some other permanent gases possess no known selective absorption. Air absorbs all the extreme ultra-violet. Fluorite is transparent throughout all the known spectrum except for two bands in the extreme infra red. Water, quartz and many silicates have strong bands in the infra red and absorb all of the ultra-violet beyond 0.2μ. Of the optical glasses, ordinary crown is opaque beyond about 320, crown beyond 300, "ultra-violet" flint transmits to about 290 and "ultra-violet" crown to $280\mu\mu$, but the absorption shades off very gradually. The various eye media are about as transparent as glass. Solutions of salts of erbium, the didymiums and related elements are unique in absorbing very narrow regions of the visible spectrum. Benzine and

similar vapors absorb narrow lines in the ultra-violet. Silver is unique among metals in having a narrow transmission band at 320 to 325$\mu\mu$.

Gases and vapors, heated or electrically excited to luminosity, have the property of absorbing those lines which they themselves are capable of emitting. If their net emission (their own emission plus other radiation absorbed and re-emitted) at any wave length is less than at adjacent wave lengths, evidently the spectrum will show a narrow darker region on a brighter ground or a *reversed* line. The Fraunhofer lines of the solar spectrum are thought to be due to absorption by the outer layers of gases of the radiation from the hotter, denser, lower layers of the sun.

Specification of Absorption.—The colors of opaque objects depend upon the ratio of reflecting to absorbing power for each wave length. *Superficial absorption* is thus complementary to reflection whether this be specular or diffuse (see Reflection) and is expressed as a fraction which is unity minus the fraction reflected.

Absorption during transmission follows the logarithmic law in every known case, that is, if a given layer absorbs a certain fraction of the transmitted radiation, the next equal layer will absorb the same fraction of what remains. If each layer of unit thickness transmits a fraction T (absorbs $1-T$), then a thickness x will transmit the fraction T^x or expressed as *Bouguer's law.*

$$I_m = I_o T_x$$

This law applies of course only to homogeneous media and monochromatic radiation. Since $T^x = e^x \log T$, and T is a fraction, we may write,

$$I_m = I_o e^{-ax}$$

where $a = -\log T$ is the limiting ratio of the percentage absorption to the thickness for very thin layers, called the *absorption coefficient.* In theoretical work the most useful specification of absorption is the absorption per wave length

called the *extinction coefficient* or absorptive index, designated by κ and defined by

$$a = 4\pi\frac{n\kappa}{\lambda}$$

where n is the refractive index and λ is the wave length in the substance. The integral of adx is a quantity frequently useful, called the *optical mass*.

The specific absorption of gases varies with the pressure (*Ångström effect*) and this, in most cases, whether the increased pressure is caused by the same or another gas.

Emission and absorption are intimately related to one another in several ways, a few of which have been formulated. A body or portion of a body can receive energy by conduction, by absorption of radiation, or internally from an electric current; it may give out energy by conduction or by radiation, hence if its temperature remains constant or varies at a certain rate, certain deductions may be drawn from the law of conservation of energy. It has long been known that bodies emit best radiation of the wave length and polarization which they best absorb. *Kirchhoff's law* states further that the emission per unit area divided by the absorption of any body at any temperature is for each wave length and polarization equal to that of a full radiator at that temperature $[E/A = R]_{T,\lambda,\phi}$. This law is of wide application, but the limits of its applicability are not yet fully known.

Reflection.

Reflection is *specular* or *diffuse* according as the reflecting surface is polished or mat. Again, reflection may occur chiefly at the surface or from layers at a considerable depth. No surface is either entirely mat or perfectly specular, but all surfaces have intermediate properties giving some diffuse and some specular reflection. A perfectly reflecting surface would be itself invisible. A pile of fine silver crystals, each facet of which is a nearly perfect mirror (specular reflector) is, as a whole, a diffuse reflector for visible radiation. Very

thin surface films may have a very pronounced effect on the reflection from surfaces. Whether specular or diffuse, reflection is *selective* or non-selective according to the equality of reflecting power for radiation of different wave lengths. All surfaces are selective reflectors if a sufficiently wide range of wave lengths is considered.

Geometrical Laws of Reflection.—In the case of specular reflection, the angle of reflection is equal to the angle of incidence and both incident and reflected rays lie in a plane normal to the reflecting surface at the point of incidence. The light path thus described is the path of least time of travel from the source to the mirror, thence to the point of observation.

The illumination of a mat surface by a beam of intensity I_0 at an angle of incidence i is equivalent (*cosine law*) to normal illumination of intensity $I_0 \cos i$. This amount multiplied by the reflecting power a, called the *albedo*, is the amount of light reflected in all directions through a hemisphere. The intensity of the light leaving the surface at an angle r will, by Lambert's law be $aI_0 \cos i \cos r/2\pi$.

Quantitative Laws of Reflection.—The reflecting power of a surface is the ratio of reflected to incident light intensity when the incidence is normal. If the refractive index of a transparent body is n its reflecting power will be

$$R = \left(\frac{n-1}{n+1}\right)^2 = \frac{I}{I_0}$$

This is a special case of the more general *Fresnel law* of reflection.

$$\frac{I}{I_0} = \frac{1}{2}\frac{\sin^2(i-r)}{\sin^2(i+r)} + \frac{1}{2}\frac{\tan^2(i-r)}{\tan^2(i+r)}$$

for light incident at an angle i and refracted at an angle r. If the reflecting surface is an interface (*e.g.*, glass-water), n is the relative refractive index of the two media. Normal reflection is the same whether the light is incident in the rarer or the denser medium. When the light is incident in the denser medium, *total reflection* will occur for angles of

incidence greater than that for which *sin* $i=n$ *sin* r. Most glass reflects about 4 percent ($n = 1.5$) normally. Some reflecting power curves are given in the accompanying figures:

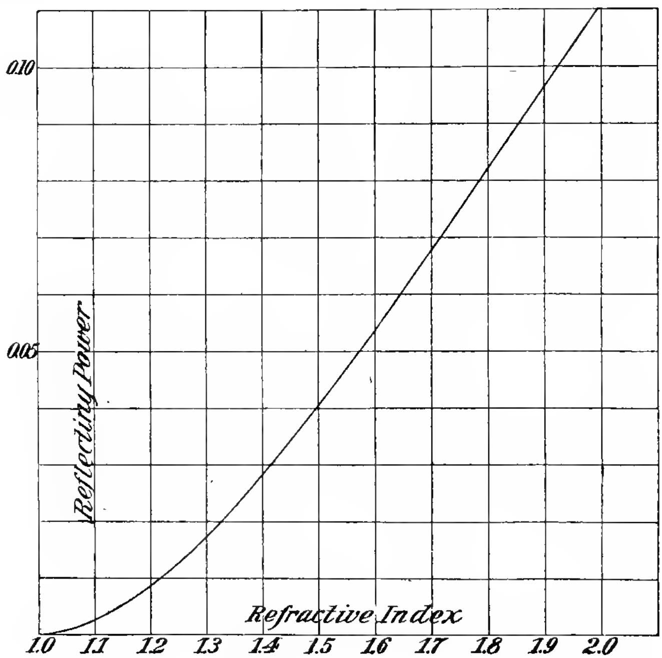

FIG. 4.—Normal reflection as a function of refractive index.

For absorbing bodies (absorptive index n) the reflecting power is given by the formula,

$$R = \frac{n^2(1+\kappa^2)+1-2n}{n^2(1+\kappa^2)+1+2n}$$

It may be noted that κ must be so large (0.1) as to produce extinction in a very small thickness (half a mm or less) before it has much effect on the reflecting power. Thus the ultra-violet absorption of glass does not appreciably increase its reflecting power. On the other hand, the high value of

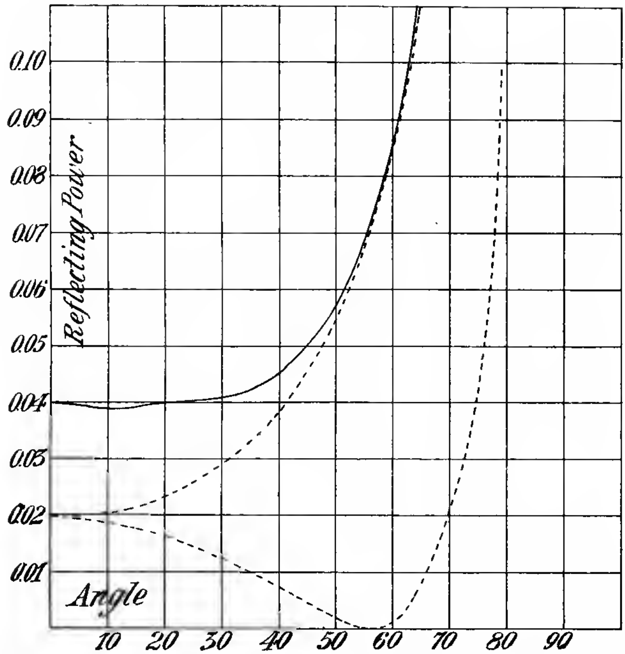

FIG. 5.—Reflecting power for n = 1.5 as a function of angle of incidence. Two components in dotted lines.

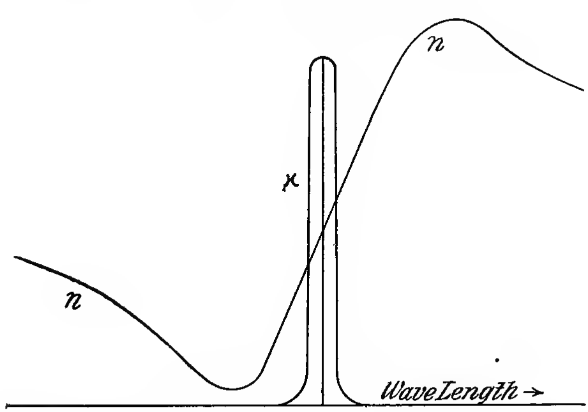

FIG. 6.—Variation of refractive index (n) in passing through an absorption band. Absorptive index χ.

κ for most metals and for glass and many salts in narrow regions in the infra-red is the chief factor in reflection. A typical case of selective absorption, and reflection is charted in Fig. 6. These curves agree well with several observed cases.

These same laws of reflection (with purely geometrical modifications) probably hold for diffuse reflection as well, but there has been very little investigation of the subject. Surfaces partly mat and partly specular often show wide variations in the proportion of diffuse to regular reflection with angle of incidence and with wave length.

Refraction.

In homogeneous media light travels in straight lines. It travels in space with a velocity of 3×10^{10} cm (186,000 miles) per second, in material substances (except a few strong absorbers) with a lower velocity. The ratio of the velocity of light in space to its velocity in a given substance is the *refractive index* of the substance. Refractive index is thus proportional to the time required for light to travel a given distance. Light travels from one point to another along that path which it can travel in least time (*Fermat's principle*). The refractive index varies with the wave length of the light used to measure it, this variation is called *dispersion*. Dispersion is *normal* when the index increases with decreasing wave length, *anomalous* when index and wave length increase together. Some typical indices are given below.

Refractive Indices for the Fraunhofer Lines.

	A.	B.	C.	D.	E.	F.	G.	H.
Glass (light crown).	1.5089	1.5109	1.5119	1.5146	1.5180	1.5210	1.5266	1.5314
Glass (dense flint) ..	1.6965	1.7070	1.7034	1.7102	1.7191	1.7272	1.7432	1.7565
Water (15° C.)	1.3284	1.3300	1.3307	1.3324	1.3347	1.3366	1.3402	1.3431
Air (0°, 760 mm.)...	1.0002893	2899	2902	2911	2922	2931	2948	2963

Refractive indices vary slightly with temperature, in gases with pressure, and in solutions with concentration.

Laws of Refraction.—(1) A ray of light incident obliquely on a smooth surface or an interface is bent or refracted. The ratio of the sine of the angle of incidence to the sign of the angle of refraction is equal to the ratio of the light velocities in the two media which is the ratio of the refractive indices of the two substances (air-glass, glass-water, etc.).

$$\frac{sin\ i}{sin\ r} = \frac{V_1}{V_2} = \frac{n_2}{n_1} = \frac{V_o/V_2}{V_o/V_1}$$

for any angle of incidence i (*Snell's law*).

(2) Incidence and refraction occur in the same plane (except for the extraordinary ray in double refraction).

Three relations between refractive index and density d have been developed.

$$\frac{n^2 - 1}{d} = const., \frac{n - 1}{d} = const., \frac{n^2 - 1}{n^2 + 2d} = const.$$

called respectively the *Newton*, the *Gladstone*, and the *Lorentz* formulas, applying to changes of index due to changes of pressure (gases) and temperature (expansion or change of state). The Gladstone formula holds well for gases, the Lorentz formula for change of state from liquid to vapor.

These formulas give refractive constants which are additive so that the refractive index of a mixture may be deduced from the indices of its components. The refractive indices of a solution for example is considered made up of the indices of the water and the *molecular refractions* of the dissolved salts. Molecular refraction in turn is considered made up of *atomic refractions* deduced from simple solutions. This additive law holds to about 1 percent in many cases.

Interference and Diffraction.

No shadow can be perfectly sharp at its margin. In passing the edge of an obstacle, the light waves are slightly bent or diffracted, the amount of diffraction depending upon the wave length of the light, distance of source to obstacle,

and of obstacle to the point of observation. Except for very narrow apertures, the diffraction is small, but it is of the utmost importance in optical instruments for it sets a limit to the resolving power of every instrument. In any image, the definition may be improved by improving the optical parts forming it, until the resolving power, determined by diffraction, is reached, but no further.

Diffraction has been worked out for but a few simple cases for the reason that each special case involves a separate investigation of the most difficult mathematical nature.

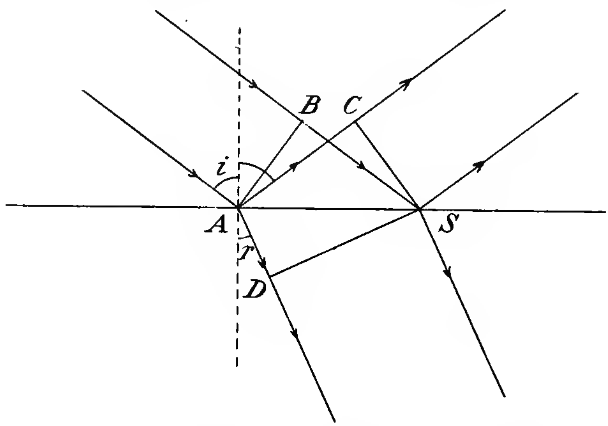

FIG. 7.—Simple refraction and reflection at a plane surface.

Kirchhoff's analytical method gives exact solutions, Fresnel's older interference method gives approximate solutions sufficient for ordinary purposes. This in turn is a development of Huyghens' geometrical method.

The light reaching any point is made up of the light arriving along all the different paths, and since these are in general not in the same phase, the resultant light at any point will depend upon the algebraic sum of the amplitudes of the light waves arriving at that point. Diffraction and refraction change both length and direction of the light

paths and hence may produce interference by bringing waves together out of step or phase. The light intensity at any point is proportional to the square of the resultant wave amplitude, relative amplitudes depending upon relative lengths of paths.

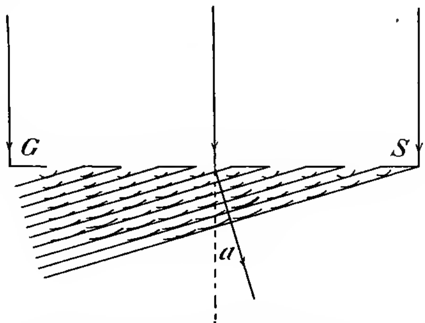

FIG. 8.—Huyghens construction for plane transmission grating, plane waves and normal incidence.

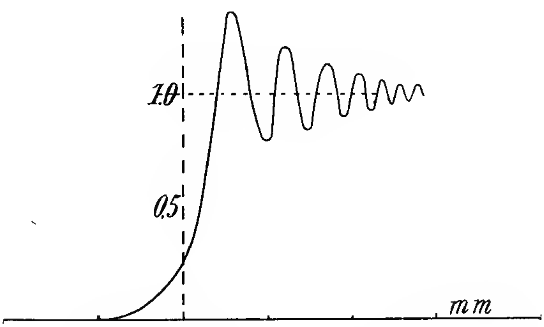

FIG. 9.—Shadow of straight edge in parallel light.

In *Huyghens'* geometrical *construction*, any wave front may be found from a preceding wave front. From each point on the known wave front as centers, circles are drawn representing small wavelets sent out. The envelope of these circles is the new wave front. Some simple cases may be dealt with in this manner. In Fig. 7 let AB represent the

plane front of a wave advancing on the surface *AS*. Then while the part *B* is traveling to *S*, *A* has traveled to *D* and

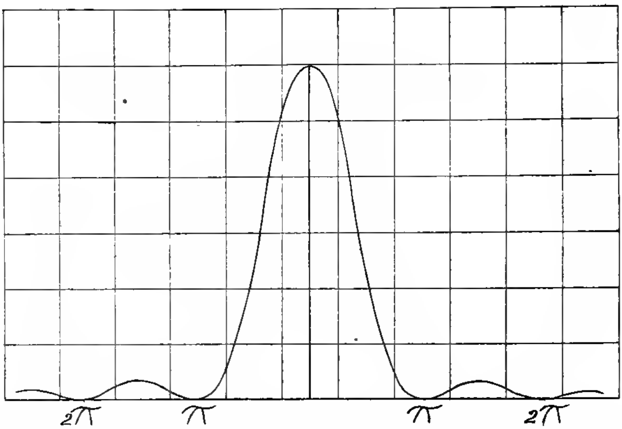

FIG. 10.—Distribution of light transmitted by a slit. Parallel incident light. Slit several wave lengths wide.

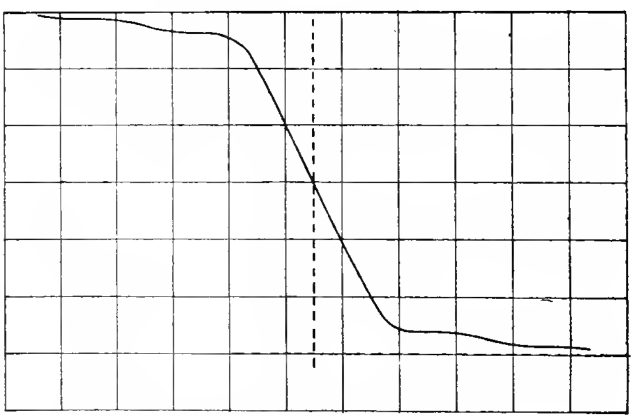

FIG. 11.—Distribution of light at edge of image of a luminous object.

to *C*, *DS* is the front of the reflected wave and *CS* the front of the refracted wave, and $\sin i : \sin r = V_1 : V_2$. In Fig. 8

let *GS* be a plane transmission grating through which plane waves (incident normally) of length λ are passing. Wavelets leaving the grating in the direction a will form a new wave front provided $d \sin a = \lambda$ or any multiple of λ.

Figures 9 and 10 are graphical representations of two important cases of diffraction. Figure 9 shows the distribution of light in the shadow of a straight edge by parallel light, the dotted ordinate representing the geometrical shadow. Figure 10 shows how parallel light passing through a slit is distributed on a normal plane.

Figure 11 shows the light distribution at the edge of the image of a luminous object, the dotted ordinate representing the geometrical position of the image. This image contains no dark bands, although the intensity does not shade off uniformly. The image of a bright point source consists of a central bright disk surrounded by alternate bright and dark rings.

Polarization and Double Refraction.

Light is supposed to consist of *transverse* vibrations rather than longitudinal, because it is capable of being polarized, that is the vibrations may occur more in one plane than in another like those of a vibrating cord. In ordinary light the vibrations are about equally distributed in all azimuths.

Most light is more or less polarized and from either of two causes. *Polarization by reflection* occurs whenever light is obliquely reflected, refracted, or emitted since the vibrations normal and parallel to the surface are reflected in unequal amounts. The polarization of the refracted light is complementary to that of the reflected light. Similarly, light obliquely emitted from a surface not mat is polarized on account of the internal reflection. Perfectly mat surfaces do not emit polarized light nor cause polarization by reflection; in fact polarized light is depolarized by reflection from a mat surface. *Polarization by double refraction* occurs in most crystals and strained solids. In these the velocity of light is different in different directions and the light wave is split

up into two complementary components plane polarized in different planes. In *uniaxial* crystals one component (*the ordinary ray*) travels with the same velocity in all directions, while the other (*the extraordinary ray*) travels with a velocity depending on the direction. Hence the ordinary index is a constant, while the extraordinary index varies in value from the ordinary index along the *optic axis* up a maximum (*positive* crystal) or down to a minimum (*negative* crystal) in a plane normal to the optic axis. In *biaxial* crystals, both rays are extraordinary, and both indices vary. They are equal in the directions of the two optic axes.

The ordinary ray lies in the plane of incidence. The wave front of the ordinary wave is perpendicular to the direction of travel, and each wavelet is spherical. The extraordinary ray does not in general lie in the plane of incidence. It advances in a direction not normal to the wave front and each wavelet is ellipsoidal. Plane polarized light is specified at any point at any instant not only by direction, intensity (or amplitude) and wave length, but by *phase* and *azimuth*. On reflection, light in general suffers a *change of phase* specified as a fraction of a wave length. In traversing a crystal obliquely to the optic axis, the components of a wave vary in relative phase, hence one component suffers a *retardation* with respect to the other. When the retardation is a quarter of a wave length or 90 degrees, the wave is *circularly polarized*. A further retardation of 90 degrees is necessary to restore the original polarization. If the retardation is not a multiple of 90 degrees, the beam will be *elliptically polarized*, the *ellipticity* being the ratio of the axes of the ellipse.

Any change in the azimuth of the plane of polarization is a *rotation*. Oblique reflection in general produces a rotation as well as a change of phase; on metallic mirrors the effect is very marked. In bodies producing rotation by transmission, plane polarized light is broken up into two circularly polarized components, one right, the other left-handed, and the rotation of the plane of polarization is due to the unequal velocities of these components.

There are three distinct classes of rotating bodies. (1) All bodies rotate in a magnetic field (*magnetic rotation*), the rotation being proportional to the component of the field parallel to the transmitted light. The rotation produced by 1 cm thickness in a field of unit strength is *Verdet's Constant*. It is for water 0.0131 degrees, for carbon bisulphide 0.0435 degrees and dense glass 0.06 degrees. (D line.)

(2) *Structural rotation* occurs in some (not all) uniaxial crystals in the direction of the optic axis. Structural rotation is lost on fusion or solution. The rotation of quartz is 21.7 degrees per mm for yellow light. Rotating crystals occur in twin forms having enantiomorphic dissymmetry of structure; that is, the two forms are images of each other, but not superposable.

(3) Many fluids and solutions show *molecular rotation* supposed to be due to enantiomorphic dissymmetry of molecular structure.

Crystals, fluids, and solutions that have rotating power are called *optically active*. All three forms of rotation are independent and superposable, and all vary with the temperature and wave length. Change of state does not affect specific molecular or magnetic rotation. Magnetic rotation is reversed by reversing the light or the field. Structural and molecular rotations are the same when the light path is reversed.

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I.

THEORY OF IMAGE FORMATION.

Optical surfaces alter the curvature of the wave front of incident light waves, hence in general change the directions of incident rays of light. Certain special forms of optical surfaces and groups of surfaces have the property of reflecting or refracting light waves and leaving spherical wave fronts still nearly spherical, and pencils of rays still nearly stigmatic pencils, and hence may form images. No system of surfaces can produce perfect images over an extended area, but sufficient perfection (for the eye or photographic plate) may be attained over a limited area to make possible a number of useful optical instruments. The *first order* theory of image formation gives the relative *size* and *position* of an extended image in relation to the optical system producing it. The *third order theory* treats of imperfections of the image called *aberrations* and their elimination.

A pencil of light is a cone-shaped bundle of rays such as would emerge from a point source or pass through a small hole. Roughly a *ray* of light is such as would pass through two small holes, a *pencil* through one small and one large hole, and a *beam* of light such as would pass through two large holes. All rays of a *stigmatic* pencil (strictly of any pencil) have a point in common. A pencil is *divergent* when the light is leaving, *convergent* when approaching the common point. Negative divergence is convergence and *vice versa.* A pencil which is nearly but not quite stigmatic is *astigmatic;* in the ordinary restricted use of the term, a pencil is astigmatic when all the rays pass through two short, straight lines instead of a point.

Simple surfaces (plane, spherical—) may reflect or refract the rays of small pencils in such a way that the bent rays

again all pass through a common point. In that case the point of convergence of the second pencil is an *image* of that from which the original pencil started. In general an image is formed whenever a pencil of light is altered in either direction or divergence in such a way that its constituent rays converge toward (or diverge from) a point other than that from which they started. Thus in Fig. 12 B and C are images of A, B is formed by a change of direction, C by a change of divergence of the rays starting from A.

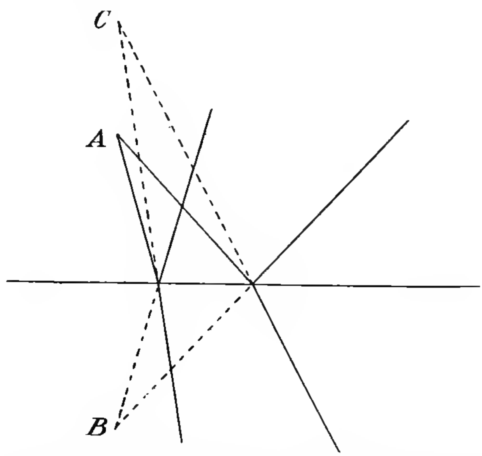

FIG. 12.—Illustrating formation of images. B by change of direction. C by change of divergence of pencil from A.

An image is *real* when formed by convergent rays, *virtual* when formed by divergent rays. Both real and virtual images may be viewed by an eye properly situated, but only real images may be received on a screen.

In forming an extended image of a large object or group of objects, an optical system makes use of two distinct systems of pencils. First, there are the *zone pencils*, cones of rays extending from each point of the object to all parts of the lens or mirror. Secondly, there are the *field pencils* converging from all parts of the object to single points in or near the

lens. Zone pencils are limited by the *entrance pupil* of the system, field pencils by the entrance *window*. Similarly leaving the optical system to form the image are the zone pencils filling the *exit pupil* and field pencils filling the exit window.

It is the prime object in the design of all optical systems to bring the null points of each of these systems of pencils into planes and as large planes as possible. The extreme types of optical systems are the microscope objective with its large zone pencils and small field pencils and the wide angle photographic objective with wide field and narrow zone pencils. Wide zone pencils give high resolving power and illumination, wide field pencils with narrow zone pencils give the pin hole effect of large flat images.

Properties of an Ideal Optical System.—An ideal optical system would transform every point in the object space into a point in the image space in such a manner that:

1. Corresponding to every object point, there is one and but one image point.

2. Object points lying on a straight line or in a plane project into image points on a straight line or in a plane.

3. Angles project into equal angles, hence (with 2) figures in space project into exactly similar figures, since we are dealing with reflecting and refracting systems only, and reflected and refracted rays lie in the plane of incidence.

4. Conjugate points lie in the same plane with an axis of symmetry.

Condition 3 cannot be realized except in the special case of no magnification. Limiting 3 to angles lying in a plane normal to the axis of symmetry, we have the set of conditions realized approximately by all optical systems.

Conditions 1, 2, and 3 determine what is known in geometry as *collinear* relationship between object space and image space. Either is obtained from the other by *homographic* projection.

From the mathematics of homographic projection there follow the well known relations between focal length, image

and object distances, magnification, etc., listed below under first order or Gauss theory, which is applicable only to rays near the axis. Hence the attempts to realize an ideal optical system are essentially attempts to make true for large pencils and large ray deviations, relations which hold exactly for infinitely narrow pencils and small curvatures.

Exact Path of a Ray.—If a ray passes the optical axis at a distance u from the apex of an optical surface and crosses it at an angle U, the deviation at this surface will cause it to again intersect the axis at a distance v and angle V given by

$$\frac{\sin i}{\sin U} - \frac{u}{R} = \frac{\sin r}{\sin V} - \frac{v}{R} = 1$$

when R is the radius of curvature of the optical surface. If the surface is a refracting surface,

$$n_1 \sin i = n_2 \sin r$$

if reflecting $r = -i$. The relation between V and U is $V + U = i - r$. If the ray passes on to a second surface

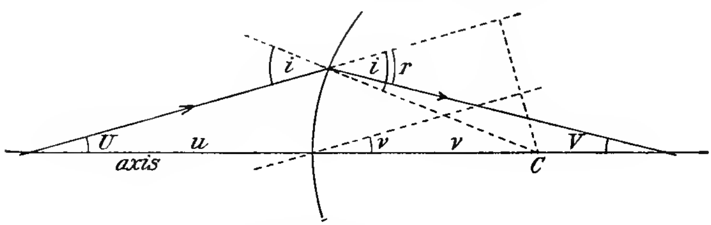

FIG. 13.—Geometrical construction showing exact path of a ray through a single spherical refracting surface.

$U_2 = V_1$ and $u_2 = s - v_1$, when s is the axial separation of the surfaces. The above equations are geometrically evident if through the intersection of the axis and the apex of the lens or mirror we draw an auxiliary line parallel to the ray and on this line drop a perpendicular through the center of curvature. Rays which intersect the axis lie in the *primary* or *meridian plane.*

Primary and Secondary Foci.—If a thin pencil is obliquely refracted at a curved optical surface, the refracted pencil

will, in general, be astigmatic. Rays lying in or near the meridian plane will come to a focus at a point nearer to or farther from the point of incidence than the secondary focal point.

Let rays from an object point O (Fig. 14) in a medium of index n, be incident on a surface at A ($OA = u$) whose center of curvature is at C and radius of curvature is R. Let AI_1

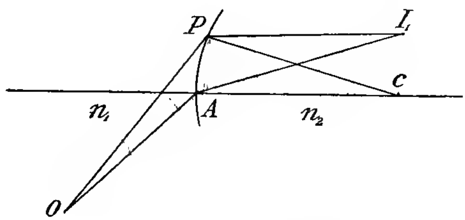

FIG. 14.—Primary image.

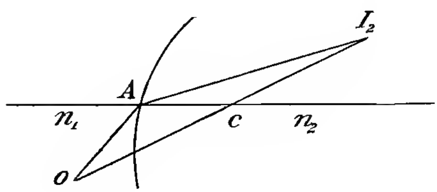

FIG. 15.—Secondary image.

($= v_1$) be the chief refracted ray and let OPI_1 be the path of an adjacent ray in the *meridian plane*. The law of refraction

$$n_1 \sin i = n_2 \sin r$$

differentiated gives,

$$n_1 \cos i \, di = n_2 \cos r \, dr$$

or $$n_1 \cos i(AOP + ACP) = n_2 \cos r\,(ACP - PI_1A)$$

hence, $$\frac{n_2 \cos^2 r}{v_1} + \frac{n_1 \cos^2 i}{u} = \frac{n_2 \cos r - n_1 \cos i}{R}$$

In a plane normal to the meridian plane (*secondary plane*), the secondary focus I_2 (Fig. 15) is at the intersection of the

chief ray AI_2 with the line of symmetry OC. The law of refraction gives,

$$n_1 \sin OAC = n_2 \sin CAI_2$$

or

$$n_1 \frac{CO}{OA} = n_2 \frac{CI_2}{AI_2}$$

But,

$$CO \cos ACO = OA \cos i + R$$
$$CI_2 \cos ACO = AI_2 \cos i - R$$

hence

$$\frac{n_2}{v_2} + \frac{n_1}{u} = \frac{n_2 \cos r - n_1 \cos i}{R}$$

For $i = 0$ this equation of the secondary image reduces to the equation for axial refraction as does also the equation for the primary image above.

If the refracting surface is spherical

$$\frac{1}{v_1} = \frac{2}{R \cos i} + \frac{1}{u} \quad \frac{1}{v_2} = \frac{2 \cos i}{R} + \frac{1}{u}$$

General Representation of Rays.—Of all the rays of the field and zone pencils but a few intersect or pass near the axis. While these rays in and near the primary plane are the ones chiefly used in lens computation, the general theory of image formation requires the representation of rays striking the lens at any point at any angle. Many systems of representation have been devised and of these, four are much used. Every such system requires the use of four independent variables.

In the *Hamilton* system used by Hamilton, Schwarzschild, Rayleigh and others in the most general theory, the axis (z), the object plane (X, Y) and image plane ($X'Y'$) are used for reference and a ray is represented by its trace (x, y) or (x', y') on one of these planes and its three direction cosines. In *Kerber's* system, vertical and horizontal planes through the (horizontal) axis are chosen for reference, and a ray's position is given by its traces on these two planes. *Seidel* and his followers use a system of polar coordinates

referred to the axis, a normal plane through the center of curvature, and the vertical plane through the axis. The position of a ray is given by the polar coordinates of its trace in the normal plane and its direction by its inclination to that plane. *Bruns* uses similar reference planes, but the notation of ordinary three dimensional geometry.

First Order Theory.

The first order theory of image formation gives the size and position of an image lying near the axis formed by narrow pencils of light subject to slight deviations throughout. The deviating system may be a mirror, a single refracting surface, a thin lens, a thick lens or a combination of lenses. The first order theory of thick lenses and lens combinations involving the idea of principal points (often called Gauss points) is due to Gauss.

Single Refracting Surface.—When the angles i, r, U and V are so small that they are sensibly equal to their sines, the exact equations give

$$\frac{n_2}{v} = \frac{n_2 - n_1}{R} - \frac{n_1}{u}$$

or written in the form of an invariant

$$n_1\left(\frac{1}{R} - \frac{1}{u}\right) = n_2\left(\frac{1}{R} - \frac{1}{v}\right)$$

n_1 and n_2 are the refractive indices of the media preceding and following the refracting surface, while u and v are object and image distances. These formulas are exact for the axis itself. The image distance v varies with the color of the light since n varies with wave length. Spherical aberration will appear later as a small correction term to v. The constant $(n_2 - n_1)/R$ is the focal length. The size ratio or magnification is the ratio $v:u$ of image distance to object distance.

Single Mirror.—If the optical surface is reflecting instead

of refracting, n_1 is replaced in general by $+1$ and n_2 by -1. In this case

$$-\frac{1}{v}=\frac{2}{R}-\frac{1}{u}=\frac{1}{f}-\frac{1}{u}$$

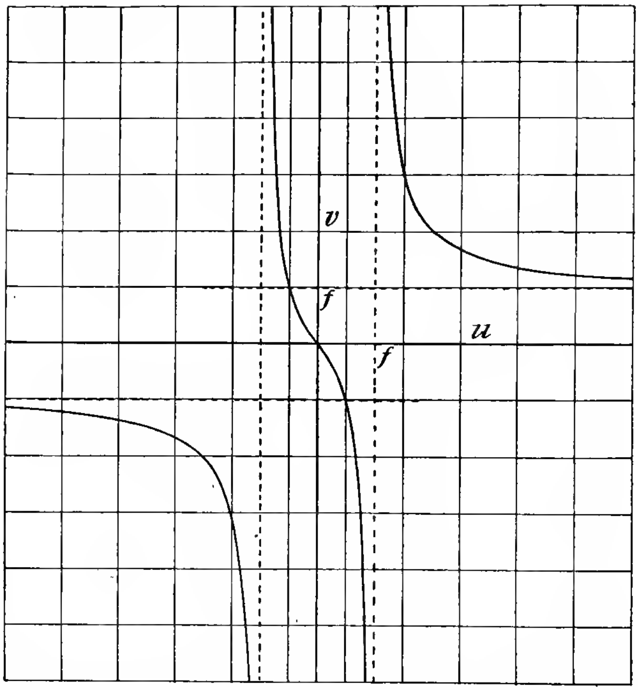

FIG. 16.—Plot of image distance (v) as a function of object distance (u) and focal length (f).

the focal length being half the radius of curvature. If the mirror is parabolized or hyperbolized, it is half the axial radius of curvature.

Two Refracting Surfaces.—The expression for the image distance in the case of two refracting surfaces in succession may readily be obtained with the relation $u_2=s-v_1$, where s is the axial separation of the two surfaces. When $n=1$

for the first and third media this expression takes the simple form for any lens

$$\frac{1}{v} = \frac{1}{f_{12}} - \frac{1}{u}$$

where

$$\frac{1}{f_{12}} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{s}{f_1 f_2}, \quad \frac{1}{f_1} = \frac{n-1}{R_1}, \quad \frac{1}{f_2} = \frac{n-1}{R_2}$$

and u and v are measured not to the refracting surfaces, but to points respectively P_1 and P_2 given by

$$P_1 = \frac{sR_1}{n(R_1+R_2) - s(n-1)}, \quad P_2 = \frac{sR_2}{n(R_1+R_2) - s(n-1)}$$

within the lens, the separation s being taken intrinsically negative for dispersive lenses. These last two expressions are the distances of the *principal points* or Gauss points from the lens surfaces. P_1 and P_2 may be either positive (measured inward) or negative (measured outward), zero or infinite according to the values of the radii $R_1 R_2$. If the third medium is not of the same index as the first (as in an eye) the Gauss points are called *nodal points*. P_1 and P_2 are the differences; equivalent focal length minus back focal length. In a *thin lens*, the separation s is negligible in comparison with the radii of curvature, hence

$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u} = (n-1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right) - \frac{1}{u}$$

and u, v, f, are measured to the lens surface. In a *thick lens*, s is the axial thickness of the lens and the preceding relations are applicable. The general relations between image distance, object distance, and focal length are plotted graphically in Fig. 16, in full lines for f positive, in dotted lines for f negative.

The equivalent focal length f_{12} written in terms of radii and index is

$$\frac{1}{f_{12}} = (n-1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right) - \frac{(n-1)^2}{n}\frac{t}{R_1 R_2} = \frac{1}{f_t}\left(1 - \frac{n-1}{R_1+R_2}\frac{t}{n}\right)$$

the last expression gives the equivalent focal length of a lens of thickness t in terms of the focal length of a thin lens of the same radii and index.

Several Thin Lenses.—If two thin lenses whose focal lengths are f_1 and f_2 are coaxial and separated by a distance s, the equivalent focal length of the combination is

$$\frac{1}{f_{12}}=\frac{1}{f_1}+\frac{1}{f_2}-\frac{s}{f_1 f_2} \text{ or } f_{12}=\frac{f_1 f_2}{f_1+f_2-s}$$

measured to the corresponding Gauss point.

The distance P_1 from the first lens to the first Gauss point is

$$P_1=\frac{sf_1}{f_1+f_2-s}=s\frac{f_{12}}{f_2} \text{ and } P_2=\frac{sf_2}{f_1+f_2-s}=s\frac{f_{12}}{f_1}$$

for the second

$$P_1 : P_2 = f_1 : f_2$$

Similarly for a combination of three thin lenses

$$\frac{1}{f_{123}}=\frac{1}{f_1}+\frac{1}{f_2}+\frac{1}{f_3}-\frac{s_{12}}{f_1 f_2}-\frac{s_{12}+s_{23}}{f_1 f_3}-\frac{s_{23}}{f_2 f_3}+\frac{s_{12}s_{23}}{f_1 f_2 f_3}$$

$$P_1=f_{123}\left(\frac{s_{12}}{f_2}+\frac{s_{12}+s_{23}}{f_3}-\frac{s_{12}s_{23}}{f_2 f_3}\right), \quad P_2=f_{123}\left(\frac{s_{23}}{f_2}+\frac{s_{12}+s_{23}}{f_1}-\frac{s_{12}s_{23}}{f_1 f_2}\right)$$

and similar formulas may be built up for any number of lenses by compounding two at a time.

Several Thick Lenses.—In case of several lenses whose thickness is not negligible (the ordinary case in practice) it is frequently of great practical importance to determine (1) the location of the pair of Gauss points for each separate lens, (2) the separation of the lenses measured between Gauss points instead of between surfaces and (3) the resultant pair of Gauss points for the whole system. The location of the Gauss points for each lens is given by the formula for a thick lens. The separation of two lenses between Gauss points is the separation between surfaces plus P_2 for the first lens, plus P_1 for the second lens. Finally the resultant pair of Gauss points for the system is obtained by compound-

ing the lenses two at a time by formulas for a pair of thin lenses, P_1 and P_2 for the pair of lenses (four Gauss points) being measured positive inward from the outer Gauss points of each of the two lenses. The two Gauss points must be distinguished since in special cases they may be coincident, reversed in relative position within a lens or system, or at infinity (telescopic system).

Geometric Construction.—If for a thick lens, parallel radii are drawn (Fig. 17), then a ray drawn to the extremity of one of these radii toward the corresponding Gauss point will leave the lens in a parallel direction at the extremity of the other radius and in line with the other Gauss point.

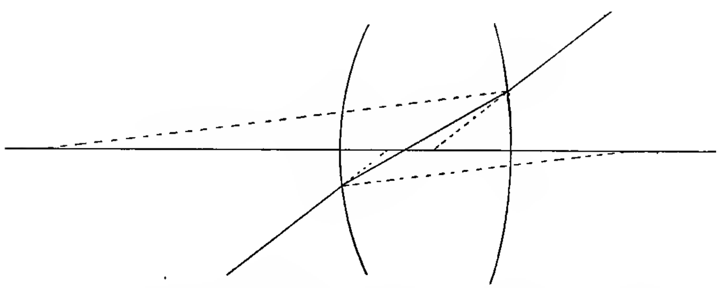

FIG. 17.—Geometrical construction for the optical center of a thick lens.

The ray crosses the axis at the *optical center* of the lens situated at a point C dividing the thickness of the lens in the ratio $R_1 : R_2$. The optical center also divides the distance between the two Gauss points in the same ratio.

In any lens combination erect planes normal to the axis at the Gauss points (Fig. 18). Then to locate the image of any object point O, draw from O one line through the principal focus, another to the first Gauss point, and a third parallel to the axis. From the intersection of the first ray with the first plane, draw a line parallel to the axis. From the second Gauss point draw a ray parallel to the second and from the intersection of the third with the second

plane draw a ray through the back focus. These last three rays all meet at the image point I, and any pair of these rays will serve to locate the image. It is customary to use the first and third.

If we connect object and image points by a direct line, this line will cross the axis at the point A which divides the separation of the Gauss points in the ratio $(u : v)$ of object distance to image distance.

The point A locates the *transverse axis* of the system, since a slight rotation of the lens system about an axis through

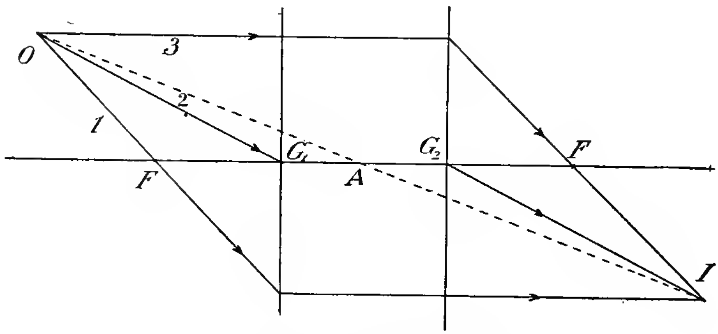

FIG. 18.—Geometrical construction for locating the image of a point object. Any optical system.

A will produce no displacement of the image. If the object is at infinity, A is coincident with the back Gauss point, while if the object is at the (front) principal focus, A coincides with the first Gauss point. These relations are very useful in the experimental location of the Gauss points and the determination of equivalent focal length. The transverse axis remains fixed out to considerable angles of obliquity in well corrected photographic objectives. In most objectives it is constant in position for all angles for which the lens is corrected and the image is good. It locates the natural stop point of a compound lens. It has no relation to the optical center of a single lens.

Third Order Theory. Aberrations.

The images formed by single lenses and by combinations of lenses not especially prepared contain gross imperfections called *aberrations*. There are seven of these aberrations of the third order, besides hybrid aberrations and aberrations of the fifth and higher orders. Of these, the seven third order aberrations, one hybrid aberration, and one-fifth order aberration are all that are considered in ordinary practice. The aberrations of second and fourth order are zero in any system having an axis of symmetry.

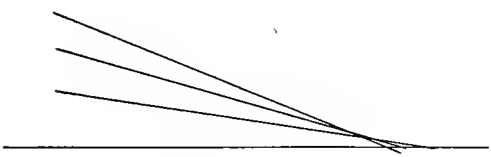

Fig. 19*a*.

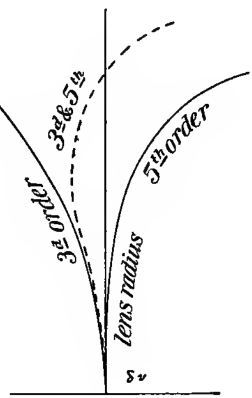

Fig. 19*b*.

FIG. 19*a*.—Pencil affected by single positive spherical aberration.
FIG. 19*b*.—Spherical aberration (δv) plotted as function of lens radius.

Three of these aberrations are due to the image-forming pencils not being stigmatic, two due to defective position of the image point and two due to the variations in refractive index with wave length. These two chromatic aberrations of course disappear in a purely reflecting system. The seven third order aberrations are as follows:

1. **Spherical aberration** is a longitudinal spreading of the rays near the image point due to different zones of the lens not having the same focal length. It varies with the vergency of the light (distance of object) as well as with the zone

of the lens. Third order spherical aberration varies with the square of the aperture and inversely as the cube of the focal length (y^2/f^3), that of the fifth order is proportional to y^4/f^5 and must be taken account of in wide zone lenses as well as the chromatic variation of spherical aberration. Spherical aberration is *positive* when rays through outer zones of the lens come to a focus nearer the lens than axial rays. It is eliminated (roughly) by balancing up with similar aberration in a negative lens. Fifth order spherical aberration is of the opposite sign from third order, but cannot be used to eliminate it since the two are far from proportional.

2. **Coma** is a one-sided blur in image points lying off the axis due roughly to one side of the lens being nearer the object than the other. In the absence of spherical aberration,

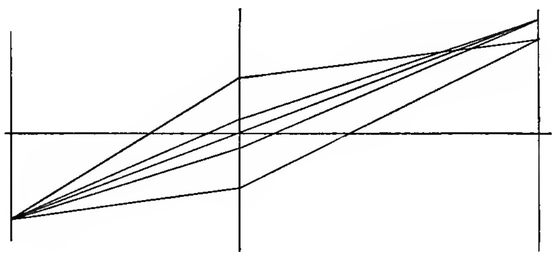

FIG. 20.—Illustration of coma.

the blur will be roughly V-shaped, directed outward if the coma be *positive*. If spherical aberration be present the blur will be rounded at both ends like a belt passing about two pulleys of unequal diameter. Coma varies with distance of object as well as with lens aperture. It is proportional to the tangent of the angle of obliquity, to the square of the aperture, and varies inversely as the square of the focal length ($:: y^2 \tan\phi/f^2$), hence does not change sign with the focal length. It may be largely eliminated, as in the earlier modern photographic objectives, by symmetry of form.

3 and 4. **Astigmatism and Curvature.**—The image of an extended plane object normal to the axis formed by a simple lens lies on two coaxial egg-shaped surfaces (Fig. 21). Radial (from axis) lines in the object will be in focus on the outer surface; tangential lines (or circles about the axis) on the inner focal surface. In other words, the image of a point in the object plane will be a short radial line on the outer image surface, and a tangential line on the inner

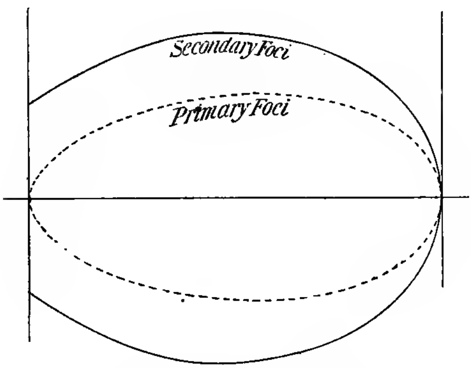

FIG. 21*a*.

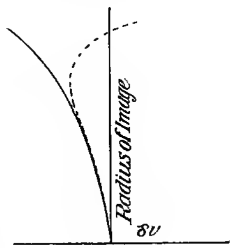

FIG. 21*b*.

FIG. 21*a*.—Curvature and astigmatism simple case. Primary foci dotted, secondary full line.
21*b*. Typical case corrected lens.

image surface. Small pencils of rays lying wholly within a primary plane will converge toward the inner, within a secondary plane toward the outer focal surface.

Correcting a lens for astigmatism consists in bringing the two image surfaces together. Correcting it for curvature of field consists in reducing the useful portion of the secondary surface to a plane by lengthening out the oblique pencils. Both astigmatism and curvature vary directly with the square of the tangent of the angle of obliquity of the rays, and inversely as the focal length ($:: tan^2\ \phi/f$). The astigmatism further varies with the vergency of the object pen-

cils, but curvature does not. Astigmatism is positive (under corrected) if, as shown in Fig. 18, the primary image surface lies within the secondary, curvature of field is *positive* if toward the lens.

5. **Distortion** is due to a variation of the magnification with distance from the axis. It is *positive* for magnification increasing with the distance from the axis.

Radial and tangential magnifications do not in general vary alike with distance from the axis, so that the complete

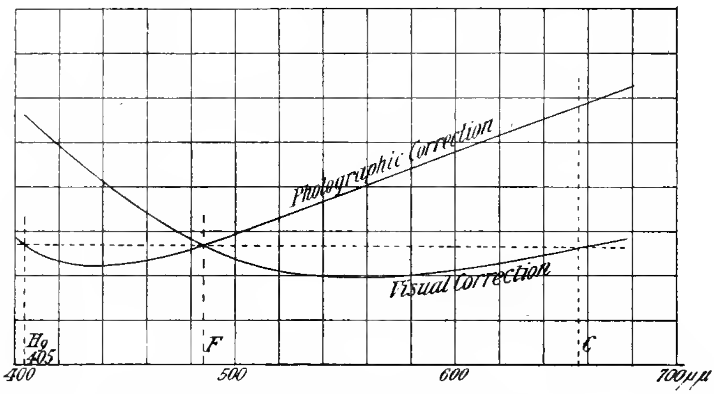

FIG. 22.—Residual variation of focal length with wave length. Two common types of chromatic correction.

specification of a distortion requires (for each object distance) two curves of magnification as a function of distance from the axis, one representing radial and one tangential distortion. The ordinary pin cushion and barrel-shaped deformation of square figures gives a rough indication of the distortion.

6. **Axial chromatism** is a variation in image distance with wave length due to variations of the refractive indices of the lenses with wave length. Objectives for visual work usually have focal lengths for the C (red) and F (blue) lines made equal for photographic work D and G' or even G' and G''. The residual chromatism, for wave lengths intermediate

between those for which the lens is corrected, is called the *secondary spectrum*. Compensation is accomplished by the use of negative lenses, since axial chromatism in single lenses takes the sign of the focal length.

7. **Lateral Chromatism.**—Lateral or oblique chromatism is a variation of sizes of images with wave length. Its elimination requires corrections quite different from those upon which axial chromatism depends. It must be carefully eliminated in oculars and lenses for three color photography.

The Development of the Theory of Aberrations.—Up to about 1840 the only aberrations that had received much attention were those most important in astronomical telescope objectives; spherical aberration, axial chromatism, the chromatic variation of spherical aberration, and to some extent coma. The theory of these had been developed by *Sir John Herschel*, *Fraunhofer*, *Gauss* and others. In the early 40's *Petzval*, of Wien, created the wide field objective, a new type of lens which gave fairly good images at some distance from the axis. In a sense, all photographic objectives are but modifications of the Petzval type. Petzval's theoretical work was nearly all accidentally destroyed, and but little is known of his theory and methods of calculation.

In the late 50's *von Seidel* (Munich) worked out the complete theory of third-order aberrations by trigonometric analysis, showed that there were five and but five of these, and stated them in terms of radii and refractive indices for any axial optical system. More recent analysis by other methods have all led to these same five aberrations, and they are called the Seidel aberrations. Since Seidel, theoretical developments have chiefly simplified details and put results in more practical form. *Abbe* developed his simple and useful sine condition for absence of coma. *Dennis Taylor* has developed the aberration theory in such form, that the roots of certain soluble equations give at once the radii necessary to reduce certain aberrations to zero. If similar equations could be developed which would give

exact solutions, theory would have made its final contribution to lens design. *Schwarzschild* and *Rayleigh* have recently obtained the five Seidel aberrations directly from Hamilton's characteristic function (1833) thus avoiding the laborious trigonometric analysis. *Whittaker* has given the simplest direct deductions of the Seidel aberrations and expressed them in the simplest form.

In the following paragraphs are given brief outlines of the methods and results of some of the best modern theories of image formation.

Theory of Transforming Surfaces.—The simplest and most general concept of an optical system is something which, receiving spherical light waves (or pencils of rays) gives them off as spherical waves (or pencils) but of different curvature. Each optical surface produces a transformation of the light wave and we require a system of such surfaces that will produce a given transformation as a whole. The series of transformations (of the variables specifying the essential groups of rays) at each of the optical surfaces forming the system constitute a transformation group and may be treated by the method of Lie's theory of transformation groups. By this method Lunn (unpublished) has obtained some of the general results of the theory of image formation. If fully developed, this theory would no doubt yield the well-known Seidel aberrations.

Theory of Paths of Equal Time.—A perfect optical system will transmit all rays traveling from a point in the object to its conjugate point in the image in equal times. Conversely, the condition that all rays of a zone pencil travel from object to image in the same time gives the conditions for a perfect optical system. This method of attacking the problem of image formation was clearly outlined by Hamilton in 1833, but remained unused until Bruns[1] in 1895 and Schwarzschild[2] in 1905 developed it and derived from it the Seidel aberrations. Rayleigh[3] (1908) greatly simplified this derivation.

Hamilton's *characteristic function* V (called the *Eikonal*

by German writers) is the path integral $\int nds$ of a light ray between conjugate points $(x'\ y'\ z')$, in the object to (x, y, z) in the image, the terminal direction cosines of the ray being $l'\ m'\ n'$ and $l\ \ m\ \ n$. The procedure is merely to write an expression for the difference between the actual path and the ideal path V and find the conditions for making this difference zero.

Following Rayleigh, let the conjugate planes be $z'=0$ and $z=0$ and consider the path difference

$$U=lx+my-V$$

Since we are dealing with axial symmetry; U must be expressible in terms of the three variables

$$x'^2+y'^2,\ l^2+m^2,\ lx'+my'$$

and since it is also small, we may write

$$U=U^{(0)}+U^{(2)}+U^{(4)}+\ldots$$

the indices referring to the degrees in which these variables enter the components of U.

$U^{(0)}$ is a constant. $U^{(2)}$ is the general second degree function,

$$U^{(2)}=1/2\ L(l^2+m^2)+M(lx'+my')+1/2\ N(x'^2+y'^2)$$

L, M, N being constants. If we stop at $U^{(2)}$ neglecting $U^{(4)}$ and functions of a higher order,

$$x=Mx' \text{ and } y=My'$$

since $x=dU/dl$ and $y=dU/dm$, and $L=0$, since (x, y) is conjugate to (x', y'). This is the simple law of magnification, neglecting distortion and other imperfections of the image.

The five Seidel third order aberrations are contained in the next term $U^{(4)}$ of the path difference U. This is of the general second degree form

$$\begin{aligned}U^{(4)}= 1/4\ A\ (l^2+m^2)^2+B\ (l^2+m^2)\ (lx'+my')\\ +1/2\ (C-D)\ (lx'+my')^2+1/2\ D\ (l^2+m^2)\ (x'^2+y'^2)\\ +E\ (lx'+my')\ (x'^2+y'^2)+F\ (x'^2+y'^2)^2\end{aligned}$$

There is no loss of generality if at this stage we put $y'=0$; that is if we assume an object point to lie on a given radial line. Hence as with $U^{(2)}$

$$x=Al\ (l^2+m^2)+Bx'(3l^2+m^2)+Cx'^2l+Ex^{3'}[+Mx' \text{ from } U^{(2)}]$$
$$y=Am(l^2+m^2)+2Bx'lm+Dx'^2m.$$

The five effective constants A, B, C, D, E represent the Seidel aberrations and are readily interpreted.

As $U^{(4)}$ contains six constants, so $U^{(6)}$ contains ten and $U^{(8)}$ fifteen, of which in each case one is ineffective.

If $A=0$, each of the remaining terms in x and y contains x' as a factor, hence if $A=0$ and $x'=0$, x and y will be 0; that is, the image will lie entirely on the axis. Further the term in A is independent of the distance x' from the axis. Therefore, $A=0$ is the condition for no *spherical aberration.*

If $A=0$ and $B=0$, there will be no *coma* or unsymmetrical differences of image not due to improper focusing. Abbe's *sine condition* is equivalent to $B=0$ regardless of whether $A=0$, since for axial conjugate points, $l'=Ml+Bl^3$, and $B=0$ makes the ratio of the terminal inclinations of the rays $l'/l=$constant. Coma is proportional to the distance x' from the axis.

If both spherical aberration and coma are absent, $dx/dl=Cx'^2$, $dy/dm=Dm'^2$, hence C and D represent departures of the primary and secondary foci from their proper plane, in fact $C/2$ and $D/2$ are the curvatures of the respective image surfaces. The condition for no *astigmatism* is then $C=D$, but unless both are zero there will be *curvature* of the image. Both astigmatism and curvature vary with the square of the distance from the axis.

Finally the condition $E=0$ is the condition for no *distortion,* varying with the cube of the distance from the axis.

The next step in the theory of aberrations would be to translate these Seidel constants in terms of radii and indices as has been done by Schwarzschild (l. c.). However, the direct derivation of the aberrations as accomplished by Whittaker

(given below) is simpler and gives further insight into their nature and relation to practical optical systems. The Hamilton theory is chiefly valuable for its breadth and generality.

Direct Derivation of the Aberrations.—Whittaker in his tract on the Theory of Optical Instruments (1907) gives direct geometrical derivations of the aberrations in terms of the optical invariants which are as simple probably as they can be derived or stated.

Consider first those conditions affecting the stigmatism of the image pencils, *i.e.*, the sharpness of the image aside from curvature, distortion, and chromatism. Suppose first that the pencils are narrow, such as would pass through a rather narrow stop. Consider any surface of the system separating media whose indices are n_1, n_2, receiving light from an object at a distance u and refracting it to an image at a distance v and from a stop at a distance a to a stop image at a distance b, the "object" in each case (except the first) being the image formed by the preceding surface.

If A_1 and A_2 represent the astigmatic differences in the pencil before and after refraction (See section on Primary and Secondary Foci)

$$\frac{n_2}{v^2}A_2-\frac{n_1}{u^2}A_1=\frac{n_2 r^2}{v}-\frac{n_1 i^2}{u}$$

i and r being the angles of incidence and refraction. If the refraction occurs at a distance y from the axis

$$i=\frac{y}{a}-\frac{y}{R},\qquad r=\frac{y}{b}-\frac{y}{R}$$

Substituting these values and using for the image and stop invariants

$$n_1\left(\frac{1}{R}-\frac{1}{u}\right)=n_2\left(\frac{1}{R}\quad\frac{1}{v}\right)\equiv I,$$

$$n_1\left(\frac{1}{R}-\frac{1}{a}\right)=n_2\left(\frac{1}{R}-\frac{1}{b}\right)\equiv K,$$

the symbols I and K, and if

$$\frac{1}{n_2 v} - \frac{1}{n_1 u} \equiv N, \text{ say,}$$

the condition for stigmatism in the final image pencil reduces to the summation (for all the surfaces)

$$\Sigma\left(\frac{K}{K-I}\right)^2 N = 0$$

This condition is known as the *Zinken-Sommer condition.*

If now astigmatism is to be absent for full pencils, the above condition must be satisfied *whatever the position of the stop*, that is I must be eliminated from it. This is accomplished by introducing the separation s between successive surfaces

$$a_2 = b_1 - s_{12},\ u_2 = v_1 - s_{12},\ v_1/y_1 = u_2/y_2$$

and similarly for all following surfaces. Summing up

$$\frac{1}{y^2(K-I)} = \sum_{p=1}^{i-1} \frac{s_p}{n_p y_p y_{p+1}} + \frac{1}{h_1^2(K_1 - I_1)}$$

the summation extending from the first surface to the i-1st. Substituting in the Zinken-Sommer condition above and writing

$$\frac{1}{I_i y_i^2} + \Sigma \frac{s_p}{n_p y_p y_{p+1}} \equiv -U_i$$

we have

$$\Sigma N\left(-I_i y_i^2 U_i + \left(\frac{y_i}{y_1}\right)^2 \frac{I_i}{K_1 - I_1}\right)^2 = 0$$

Since in this K_1 is the only quantity involving the position of the stop, it will be satisfied if the coefficients of the various powers of $\frac{1}{K_1 - I_1}$ are separately zero, *i.e.*, the optical system

will give point images at full aperture provided it satisfies the three conditions

(I) $$\sum_i I_i^2 y_i^4 N_i = 0$$

(II) $$\sum_i I_i^2 y_i^4 N_i U_i = 0$$

(III) $$\sum_i I_i^2 y_i^4 N U_i^2 = 0$$

known as Seidel's first, second, and third conditions.

Condition (I) taken alone represents the condition that the system shall give point images by all pencils which can pass through a stop at the axial point $K_1 - I_1 = 0$ of the object, hence that there be no spherical aberration in the narrower sense. Similarly (II) represents the condition for no coma and (III) for absence of astigmatism. If the Zinken-Sommer Condition is satisfied, (III) amounts to the condition for flatness of field

(IV) $$\sum_i \frac{1}{R_i}\left(\frac{1}{n_i} - \frac{1}{n_i - 1}\right) = 0 \equiv \sum_i P_i \textit{ say,}$$

known as *Petzval's condition.* This for thin lenses in air reduces to $\Sigma(1/nf) = 0$.

The condition for the absence of distortion is obtained from the condition that the products of the magnification ratios for all the refracting surfaces shall be independent of the position of the object point in the object plane and shall be further independent of the presence or position of any arbitrary stop. These two conditions may be written in the form

(V) $$\sum_i (V_i U_i^3 + P_i U_i) = 0$$

where

$$V_i \equiv I_i^2 y_i^4 N_i$$

Summing up, the condition for the absence of

Spherical aberration is	$\Sigma V_i = 0$
Coma	$\Sigma V_i U_i = 0$
Astigmatism	$\Sigma V_i U_i^2 = 0$
Curvature of Field	$\Sigma P_i = 0$
Distortion	$\Sigma (V_i U^3 + P_i U_i) = 0$

it being assumed in each case that all of the preceding conditions have been satisfied.

The above expressions represent a complete solution to the third order of the problem of image formation from the academic standpoint. Given the refractive indices and radii of curvature of each component of a system, they give the conditions for the elimination of any third-order aberration for any given distance of object. Given further the dispersions of the component glasses, the focal and oblique chromatic differences may easily be calculated from the indices for any two colors or wave lengths. But for practical purposes these Seidel conditions are almost useless. Even if transformed so as to give the amount of residual aberration in any case, this residual could be better obtained by the direct method of computing through particular rays. Nor can these transformed expressions for the aberrations be treated as simultaneous equations and solved, for in them the pencil vergencies (given by u and v) as well as the aberrations, are active functions of the radii and separations regarded as unknowns.

We are indebted to Dennis Taylor for first (1906) deriving or at least publishing soluble expressions for the aberrations. These still contain u and v as functions of the radii, but in such form that they are nearly constants, and the equations give useful first approximations. The application of these equations to lens design will be discussed in the following chapter. They are derived from the consideration of individual rays. As their derivation is not here of particular interest, they are merely given in final form below.

Soluble Aberration Equations.—In Taylor's system the *shapes* of the component lenses are left for final variables, the roots of the equations to be solved. A variable x is introduced to represent this shape (following Coddington), such that

$$1+x=\frac{2(n-1)f}{R_1},\quad 1-x=\frac{2(n-1)f}{R_2},\quad x=\frac{R_2-R_1}{R_2+R_1}$$

also

$$1+\alpha = \frac{2f}{u},\quad 1-\alpha = \frac{2f}{v},\quad \alpha = \frac{v-u}{v+u}$$

and

$$1+\beta = \frac{2f}{a},\quad 1-\beta = \frac{2f}{b},\quad \beta = \frac{b-a}{b+a}$$

As x can be varied without varying either the focal length f of a component or the separations, u and v are nearly though not quite independent of x. The quantities a and b (as above) correspond to u and v, but refer to the field pencils instead of the zone pencils, that is, they refer to rays passing through the transverse axis or center of the stop if there be one. The introduction of the auxiliary variables x, α, β greatly simplifies the equations.

In order to clearly bring out the forms of these aberrations for multiple systems, they are here given for a system of four lenses. The law of formation is evident and they may be readily written down for fewer or more components.

Third Order Aberrations, Taylor's Forms.

Spherical Aberration

$$\delta\frac{1}{v} = \frac{y_1^2}{8}\left[\frac{A_1}{f_1^3}\left(\frac{v_1v_2v_3}{u_2u_3u_4}\right)^2 + \frac{A_2}{f_2^3}\left(\frac{v_2v_3}{u_3u_4}\right)^2\left(\frac{u_2}{v_1}\right)^2 + \frac{A_3}{f_3^3}\left(\frac{v_3}{u_4}\right)^2\left(\frac{u_2u_3}{v_1v_2}\right)^2 + \frac{A_4}{f_4^3}\left(\frac{u_2u_3u_4}{v_1v_2v_3}\right)^2\right]$$

$$\text{where } A \equiv \frac{n+2}{n(n-1)^2}x^2 + 4\frac{n+1}{n(n-1)}\alpha x + \left(3+\frac{2}{n}\right)\alpha^2 + \frac{n^2}{(n-1)^2}$$

Coma

$$\tfrac{3}{4}\tan\phi\, y_1^2\left[\frac{1}{f_1^2}\left(\frac{A_1}{\alpha_1-\beta_1} - C_1\right) + \frac{1}{f_2^2}\left(\frac{A_2}{\alpha_2-\beta_2} - C_2\right)\left(\frac{u_2}{v_1}\right)^2 + \frac{1}{f_3^2}\left(\frac{A_3}{\alpha_3-\beta_3} - C_3\right)\left(\frac{u_2u_3}{v_1v_2}\right)^2 + \frac{1}{f_4^2}\left(\frac{A_4}{\alpha_4-\beta_4} - C_4\right)\left(\frac{u_2u_3u_4}{v_1v_2v_3}\right)^2\right]$$

$$C \quad \left(2+\frac{1}{n}\right)\alpha + \frac{n+1}{n(n-1)}x$$

Eccentricity

$$\Sigma \frac{tan^2 \phi}{2f} \frac{1}{\alpha-\beta}\left(\frac{A}{\alpha-\beta} - 2C\right) \text{ in secondary plane}$$

3 times the above in primary plane.

Curvature

$$\tfrac{1}{2}\Sigma \frac{tan^2 \phi}{f} \frac{3n+1}{n} \text{ in primary plane.}$$

$$\tfrac{1}{2}\Sigma \frac{tan^2 \phi}{f} \frac{n+1}{n} \text{ in secondary plane}$$

Distortion

$$tan\ \varepsilon = tan\ \phi + a_1^2\ tan^3 \phi$$

$$\left[\left(T_1 + \frac{1}{f_1}\frac{B_1}{\alpha_1-\beta_1}\right) + \left(T_2 + \frac{1}{f_2}\frac{B_2}{\alpha_2-\beta_2}\right)\left(\frac{a_2}{b_1}\right)^2 + \dots \right.$$

$$\left.\left(T_3 + \frac{1}{f_3}\frac{B_3}{\alpha_3-\beta_3}\right)\left(\frac{a_2 a_3}{b_1 b_2}\right)^2 + \left(T_4 + \frac{1}{f_4}\frac{B_4}{\alpha_4-\beta_4}\right)\left(\frac{a_2 a_3 a_4}{b_1 b_2 b_3}\right)^2\right]$$

$$T = \frac{n+1}{n(n-1)}x + \frac{\beta}{n}$$

$$B(n\ x\ \beta) \equiv A(n\ x\ \alpha)$$

Axial Chromatism

$$\delta\frac{1}{v} = \frac{1}{f_1}\frac{\Delta n_1}{n_1-1}\left(\frac{v_1 v_2 v_3}{u_2 u_3 u_4}\right)^2 + \frac{1}{f_2}\frac{\Delta n_2}{n_2-1}\left(\frac{v_2 v_3}{u_3 u_4}\right)^2 +$$

$$\frac{1}{f_3}\frac{\Delta n_3}{n_3-1}\left(\frac{v_3}{u_4}\right)^2 + \frac{1}{f_4}\frac{\Delta n}{n_4-1}$$

Oblique Chromatism

$$\frac{2}{\alpha_1-\beta_1}\frac{\Delta n_1}{n_1-1} + \frac{2}{\alpha_2-\beta_2}\frac{\Delta n_2}{n_2-1} + \dots$$

We have now traced the theory of image formation from its simplest and most general form, through various stages down to expressions for defects in the image in terms of the language of optical engineering. In the following chapter the application of this theory to practical optical design will be outlined. Anyone who has ever entered the field of optical engineering knows that it is neither easy nor simple and yet how interesting and full of possibilities its difficulties

and problems are. Neither the theory nor the applications have yet by any means been fully developed.

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II.

DESIGN AND TESTING OF OPTICAL SYSTEMS.

While the properties of any optical system may be determined from its constants by calculation without any serious difficulty, the inverse problem of designing a system free from errors is very much more difficult and is at best still somewhat of a matter of cut and try. In the solution of a set of equations such as Taylor's, certain small quantities are neglected and it is only by the exact computation of rays through a system that its properties can be precisely determined. Finally an experimental test of the system must be made to determine the coordination of its properties and residual errors of design and manufacture. This chapter is an outline of the principles and methods commonly used in designing and testing optical systems. These are given in broad terms, for each individual in any case will develop details to suit his own tastes and convenience.

An algebraic analysis of the general problem of design gives at once the theoretical maximum of attainable corrections. In a system having N component lenses there are at disposal;

$$\left.\begin{array}{l}N \text{ focal lengths}\\ N \text{ lens forms}\end{array}\right\}\text{ or } 2N \text{ radii.}$$

N glass indices
N glass dispersions
N thicknesses
$N-1$ separations
$-S$ cemented surfaces in contact,

the glass indices, dispersions and thicknesses available being to some extent limited by the limitations of glass manufacture. Altogether then there are $6N-1-S$ independent

quantities at our disposal in designing any optical system. Each condition imposed on the system will use up one and but one of these $6N - 1 - S$ degrees of freedom.

The conditions to be satisfied cannot be stated so definitely, as they are more or less matters of choice and to some extent interdependent. These conditions are in full:

Aperture ratio, 1 condition

Spherical aberration (including all orders) = 0 for Z_1 zones C_1 colors and D_1 distances of object, $C_1 + Z_1 + D_1$ conditions.

Coma = o for A_2 angles of obliquity and D_2 object distances, $A_2 + D_2$ conditions.

Astigmatism = o for A_3 angles of obliquity and D_3 object distances, $A_3 + D_3$ conditions.

Curvature = o for A_4 angles of obliquity and D_4 object distances, $A_4 + D_4$ conditions.

Distortion = o for A_5 angles of obliquity and D_5 object distances, $A_5 + D_5$ conditions.

Axial chromatism = o for C_6 colors and D_6 object distances, $C_6 + D_6$ conditions.

Oblique chromatism = o for C_7 colors A_7 obliquities and D_7 object distances, $A_7 + C_7 + D_7$ conditions.

to which must be added special conditions, for instance that either Gauss point be in a given position as is often required.

The total of all these conditions is a large number for a system of even moderate excellence, so that in any case a large proportion of them must be sacrificed to bring the number within $6N - 1 - S$. The variation of each aberration with distance of object is usually so small that for most systems $D = 2$ throughout is sufficient. Oblique aberrations are usually reduced to zero for one, sometimes two additional wave lengths. Spherical aberration is reduced to zero usually for but one zone about four-fifths of the lens radius out. Hence if all aberrations are given equal weight at least seventeen conditions must be satisfied to secure even rough correction throughout. The older practice was to secure

corrections for the more important aberrations and to ignore the rest, but the modern tendency is toward placing all the aberrations on an equal footing. Thus we have telescope objectives corrected not only for spherical aberration, axial chromatism, and coma, but for curvature, oblique chromatism and distortion.

If but 17 conditions were to be satisfied and the 17 equations in $6N-1-S$ variables could be set up, it will be seen that the algebraical difficulties would be enormous. These, however, are insignificant compared with the labor involved in cut and try methods.

Quite a number of these conditions have been given special names. The five *Seidel conditions* were given in the previous chapter. Seidel's second condition (no coma) he called the *Fraunhofer condition* because it was found to be satisfied by a celebrated telescope objective designed by Fraunhofer years before. It may be derived from and is equivalent to Abbe's *Sine condition*. A system which is free from spherical aberration and also satisfies the sine condition is called *aplanatic*. *Herschel's condition* is that if a system be freed from spherical aberration for one object distance, it shall remain free for a slightly nearer or more distant object. It is equivalent to the second Seidel condition in two special cases; no magnification and for object at infinity and system telescopic, in all other cases it is incompatible with this condition. As developed by Herschel it included only third order aberrations. It was later developed by Abbe in a general form not limited to the third order. The *Zinken-Sommer condition* is that image points be sharply defined though they may not lie in a plane. It is equivalent as we have seen, to the first three Seidel conditions.

The *Gauss condition* is generally taken to mean the absence of chromatic variation in the correction for spherical aberration. Gauss made three focal lengths equal; axial rays of two different wave lengths and an edge ray of one of those wave lengths. *Petzval's condition* for flatness of field is equivalent to the fourth Seidel condition. That there be no

spherical aberration for rays passing through the center of the stop point is sometimes called *Lummer's condition* for no distortion. It is part of Seidel's fifth condition (absence of distortion). Systems that are free from secondary spectrum and aplanatic are called *apochromatic*, while if free from spherical aberration for more than one color yet not entirely free from secondary spectrum they are called *semi-apochromatic*.

The conditions to be fulfilled by any optical system may be thus summarized:

1. *Zone-pencils*, of all apertures, for any object distances and wave lengths for which the system is to be used, must be brought to the same focal point.

2. All *field pencils* for all distances of object plane and all wave lengths for which the system is to be used must come to a focus in the same plane.

3. *Magnification* ratios must be preserved for all field angles, magnifications and wave lengths for which the system is intended.

In other words, the system must be free (within required limits) from both *zonal and chromatic differences for rays from an axial object point and for rays through the transverse axis.*

There are certain physical and practical limitations to the stigmatism of image pencils. Rays cannot be and need not be in any case brought to a mathematical point. Defraction and interference of the light waves near the apex of a pencil, limiting the resolving power of the objective, make an exact union of the rays impossible. Secondly, such an exact union would be useless if possible. In visual systems stigmatism need be carried no farther than what would correspond with the resolving power of the eye and structure of the retina. In photographic systems, the limit is fixed by plate grain.

Practical Design.

The practical design of any system falls naturally into three distinct steps: (1) *rough design:* choice of number of

component lenses, selection of glass, choice of focal lengths and separations of components; (2) determination of individual radii by the *solution of equations* to obtain stigmatic image pencils; (3) final *retouching and testing* by differentials and the calculation of selected individual rays through the system.

The expressions of the aberrations developed by Taylor, given at the close of the last chapter, are a guide to the order in which the aberrations should be considered. The simplest is that for curvature, depending only upon the focal power and indices of the component lenses. Next in order come axial and oblique chromatism. None of these three involves the form of a lens. In the first step all of these interdependent conditions must be considered together to set up a tentative system of the required aperture ratio and number of components. In the second step, equations for spherical aberration, coma, astigmatism, distortion, and the Gauss condition are set up and solved for the shape x of each component lens. This gives the approximate form of the tentative system, the outstanding errors being chiefly those of the fifth order. Finally a set of rays is triangulated through the lens as a test and guide to final retouching. If the ray residuals are large they are entered as constants in the equations and the second step repeated.

1. **Rough Design.**—The first matter to be decided is the number of component lenses. This depends upon the aperture required, and field to be covered. For aperture 0.05 ($f/10$) and field angle 0.1 (radian) two separated components (Clark type) or three cemented components are sufficient to secure definition equal to the resolving power and good achromatism as well. The number of components increases with both aperture and field. Three separated components is sufficient (Cooke type) for apertures up to about 0.2 and fields of about 1.2. For still wider apertures (high speed and high resolving power objectives) and wider fields (wide-angle objectives) more components used. Both aperture and field are kept down to minimum

requirements not only to simplify the aberration equations by reducing the number of necessary components, but to avoid the troublesome fifth order aberrations. These as we have seen, do not lend themselves to algebraic treatment and though they do yield to cut and try methods, these are extremely laborious.

During the decade in which were produced most of the modern anastigmats (1895–1905), the procedure was roughly this: first, a good telescope objective, either a separated doublet of the Gauss or Clark type or a cemented triplet was computed free from spherical aberration and axial chromatism. This objective was then doubled to form a symmetrical four or six piece photographic objective symmetrical about a central stop. This doubling gave high speed by nearly doubling the numerical aperture, and eliminated nearly all the coma, distortion and oblique chromatism. At the same time curvature errors were greatly reduced by the introduction of the new type of glass of high index and low dispersion.

However, Taylor showed, and it is apparent from his equations, that in securing wide aperture and field a lens should be symmetrical not with respect to a central point, but with respect to the light beam in terms of $\alpha - \beta$, f, and relative dispersion. Taylor further showed that in a separated system it is sufficient to only approximately satisfy the Petzval condition, and that the use of the new glass type is not essential in eliminating curvature errors. This is because a separated system may gain a considerable power from the separation alone, that is $\Sigma\ 1/f$ need not be zero, and yet the curvatures be small relative to the focal power of the system, for the curvature is independent of the separation. Since 1905 a number of highly corrected unsymmetrical objectives have been developed.

After deciding upon the aperture to be employed and the number of component lenses, the next step is to choose the focal lengths of the components to satisfy the Petzval condition, either closely or roughly according as the system

is to be a cemented or uncemented one; next select glasses and separations to satisfy the two chromatic conditions

$$\Sigma \frac{1}{\nu f} \mathrm{II} \left(\frac{v}{u}\right)^2 = 0 \text{ and } \Sigma \frac{1}{\nu(\alpha - \beta)} = 0$$

This requires skill and patience but must be accomplished as completely as necessary at this point, as conditions cannot be altered later. If the system is to be achromatized for three colors, the chromatic conditions must be repeated to cover the third color.

2. **Solution of Equations.**—Having determined the focal lengths, indices and separations between Gauss points of the component lenses in a system, the next step is to set up and solve the equations giving the exact form of each lens. All the remaining aberrations (spherical aberration, coma, astigmatism, and distortion) are functions of the several x's. The several n's and α's have been determined and β is determined from the position of the transverse axis. These four aberrations give four equations in as many variables (x_1 x_1^2 x_2 x_2^2 . . . .) as there are component lenses. If the Gauss condition is to be imposed, another spherical aberration equation in the altered index or a derivative must be added.

We have, then, finally for solution either three, or four, or five simultaneous quadratic equations with numerical coefficients in as many variables as there are lenses. If there are more variables than equations, further conditions may be imposed, if a certain surface be cemented or a certain lens is made symmetrical or plane on one side or like another lens, one variable disappears. If there turn out to be more equations than variables, a least square solution may be made giving the minimum residual aberrations.

Roots greater than unity (absolute) indicate meniscus lenses; if these are not desired, other roots must be sought less than unity. For a symmetrical lens $x=0$, for a plano-convex or -concave lens $x, = +1$ or -1 according to whether the light first strikes the curved or flat surface.

If roots are imaginary (unless nearly zero) or very large (indicating deep curves) the whole system must be rejected, but a system which satisfies the first set of conditions (step 1) will generally be found to give real roots lying between 0 and ± 1.

3. **Computation of Rays.**—Having determined the lens forms given by the roots of the equations in x_1 x_2 ...., these are translated into radii and the lens system laid off in exact form from the Gauss points. The axial thicknesses of the lenses are made roughly for positive lenses $\sqrt{2}$, for negative lenses $1/\sqrt{2}$ times the thickness where transversed by the edge ray. The next step is to compute a set of rays through the system as a test and as a guide to the elimination of fifth order defects and the third order thickness effects neglected in step 2.

The first set of rays to be computed through are those from an axial point of the object. Of these are chosen the axial ray and the edge ray, with intermediate rays striking the first lens at distances $\sqrt{1/4}$, $\sqrt{2/4}$, $\sqrt{3/4}$ of the (aperature) radius from the axis. These quadratic ratios are chosen because the third order spherical aberration, coma, and distortion vary with y_1^2 and by taking the initial rays at these distances the computed v's can be easily checked by differencing at any point in the system; the presence of fifth order aberrations is at once apparent from a lack of equality in the v differences. The axial ray is carried along with the others (by the axial formula) as a control and a further check against errors.

Rays are computed by the exact formulas given below. The problem is to obtain the image intercept v from the object intercept u for each surface. The necessary steps are, in order:

u object intercept,
$\sin i$ angle of incidence,
$\sin r$ angle of refraction,
V elevation of image ray,
v image intercept.

Let U be the elevation of the object ray and R the radius of curvature of a surface, then the complete formulas are:

$$1.\ \sin i = \sin U \left(\frac{u}{R} \pm 1\right)$$

$$2.\ n_2 \sin r = n_1 \sin i.$$

$$3.\ V = i - r \pm U.$$

$$4.\ v = R\left(\frac{\sin r}{\sin V} \pm 1\right).$$

Then U for any surface is the same (in magnitude) as the V of the preceding. Rays are started in the first surface by simple special formulas, either

$$\tan U = y/u \qquad \text{or}$$
$$\sin i = y/R$$

the first if the object is taken at a finite distance u_1, the second if the entering rays are parallel to the axis, $u_1 = \infty$.

The question of algebraic sign is a troublesome one in 1, 3, and 4. Consistent systems of signs have been worked out, advocated and no doubt used, but hardly any two systems are alike and probably the best method is to use a sketch control diagram for each surface. This shows at a glance geometrically which is the proper sign and in time the computer will develop his own conventions as to sign.

Finally consider in detail all the steps in the computation for any surface, say the fourth to be concrete. The steps in order are

$u_4 = s_{34} - v_3$

u_4/R_4

$u_4/R_4 \pm 1$

$$\sin i_4 = \sin V_3\left(\frac{u_4}{R_4} + 1\right)$$

$\sin r_4 = n \sin i_4$ or $\sin i_4/n$

i_4 and r_4 from their sines

$V_4 = i_4 - r_4 \pm V_3$

$\sin V_4$

$$\sin r_4/\sin V_4$$
$$\sin r_4/\sin V_4 \pm 1$$
$$V_4 = R_4(\sin r_4/\sin V_4 \pm 1)$$
$$u_5 = S_{45} - v_4$$
(etc, etc.).

At the same time the axial ray is brought forward by the formulas

$$\frac{n_2}{v_4} = \frac{n_2 - n_1}{R_4} - \frac{n_1}{u_2}$$
$$u_4 = S_{34} - v_3$$

in which the signs of u, v, and R are those of the first order theory. This is a very useful control formula for the oblique rays. For example a certain set of v's were

	v	$\delta_1 v$	$\delta_2 v$
axis	121.49		
rad./2	120.62	0.87	0.00
rad.$\sqrt{2}$	119.75	0.87	0.01
rad.$\sqrt{3}/2$	118.89	0.86	0.03
edge	118.06	0.83	

with their first and second differences. Each v has apparently been carried through without error and there is a slight but perceptible fifth order aberration indicated by the second differences.

When a surface is nearly or quite flat (R large) or a ray runs nearly parallel to the axis, special formulas free from quantities approaching 0 or ∞ are required. Various substitute formulas are given by Czapski, Steinheil and Voit, Gleichen, von Rohr and others. For R large (4) has been put by Tillyer in the convenient form free from R

$$v = u\frac{\sin U}{\sin V}\,\frac{\cos 1/2(V+r)}{\cos 1/2(U+i)}$$

When either V or U are small, it is usually sufficient to treat data temporarily as exact to two more decimal places until the rays again diverge from the axis.

For most work it is sufficient to carry five figures in the computation, but when extreme precision is required or the rays at some point become nearly parallel to the axis, seven are necessary. In computation, every possible saving of labor or confusion should be made. For computing, some prefer logarithms, including addition and substraction logarithms of the Gauss type, while others prefer a computing machine in conjunction with a table of sines of decimal angles. Either is much simpler than ordinary logarithmic computation and more precise than work with a slide rule.

The test for the presence of *spherical aberration* of any order is of course the departure from equality of the final v's. The test for fulfillment of the *sine condition* is made by dividing each initial *sin* U (or y_1 if U_1 equals 0) by the corresponding final *sin* V. Departures from equality give a warping of the surface in which initial and final rays intercept. The final axial v is the *back focal length* of the system. The *equivalent focal length* is the back focal length plus the distance from the back Gauss point to the back surface.

Residual axial chromatism and fulfillment of the *Gauss condition* for any second or third wave length is obtained by computing through additional rays with corresponding modified indices.

Finally a third set of rays passing through the transverse axis in the various required directions computed for both primary and secondary planes, give the residual *astigmatism, curvature and distortion* (*tan* V/*tan* U). A second set of rays computed for the modified indices gives the residual *oblique chromatism.*

Residual Errors.

In every image formed by an optical system there is a certain amount of diffuseness or lack of definition, shown near sharp points or edges if of sufficient contrast. This shading in an image may be so slight as to be microscopic, but it is always there. Some of it is unavoidable and some

may be due to aberrations not eliminated. We have next to analyze and classify this lack of definition, discuss methods of observing, measuring, and specifying it, and assigning upper limits to it in various optical systems.

Resolving Power.—If an image is formed by light of wave length λ by means of a system the radius of whose entrance pupil is r, then

$$\phi = a\frac{\lambda}{r}$$

is the angular separation of objects just separated in the image. The constant a depends upon the quality of definition in the image. In an object just barely apparent a equals 0.4 when it is just possible to identify the object $a=0.6$, while $a=0.9$ or $a=1.0$ corresponds[1] to a clearly defined image. If the object is a double star, then the image of one lies in the first dark diffraction ring surrounding the other when $a=0.61$.

Since angular separation in object and image is the same, if the object is at either the principal focus or infinity, we may replace ϕ in the above by δ/F where F is the focal length of the objective and δ the linear distance between the objects to be resolved. The above formula then becomes

$$\frac{\delta}{\lambda} = a\frac{F}{r}$$

hence for an image (or object) just resolvable ($a=0.5$), *the resolved detail measured in wave lengths is equal to the aperture ratio* ($F/2r$) of the objective. In a microscope objective of numerical aperture (n_1 *sin* U_1) $=0.5$, δ is approximately one wave length (0.00044 mm.) and as a matter of experiment, rulings on glass 50,000 per inch (Grayson rulings) can just be resolved by such an objective.

This quantity δ represents the irreducible minimum of diffusion attainable with any system however perfectly corrected so long as it operates with light waves. The limitation to its value is due fundamentally to the inter-

ference of these light waves (see Fig. 23). Tests of resolving power are easily made with a half tone screen of say 200 lines per inch illuminated with monochromatic light. The image must of course be viewed with an ocular or microscope of sufficient power to give retinal images not too fine for the resolving power of the eye.

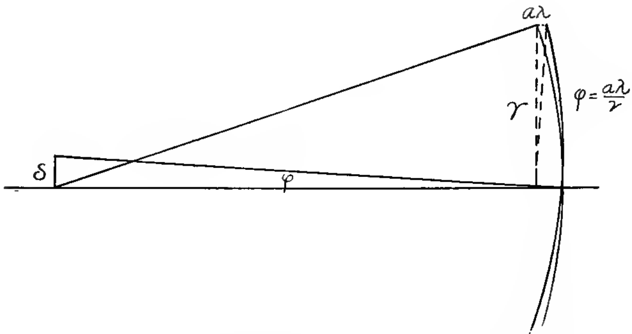

FIG. 23.—Resolving power, aperture and wave length.

Good telescope and microscope objectives give, at full aperture, definition fully up to their computed resolving powers. Good photographic objectives give such definition only when stopped down to half or one-third full aperture.

In a system composed, say of a condenser, an objective, an ocular, and an eye, suppose it is desired to realize the full resolving power of the objective. For equal resolution throughout

$$\phi = \frac{\delta_1}{u_2} = \frac{\delta_2}{v_1}, \ \frac{\delta_2}{u_2} = \frac{\delta_3}{v_2}, \ etc.$$

which is merely the condition that each successive component of the system have sufficient aperture to admit the full pencil transmitted by the objective.

Suppose on the contrary it is desired to design a system (objective and ocular say) which shall utilize the full resolving power of the eye. Taking the diameter of the eye pupil as

4 mm, corresponding to a moderate illumination, and equivalent focal length in air 15.0 mm, the aperture ratio is 3.7. For yellow light ($\lambda = 6\mu$), the smallest detail at the retina that the eye can resolve is 3μ or 0.003 mm, approximately the distance between centers of adjacent rods and cones. If an ocular of focal length f_c be before the eye, its aperture ratio should therefore be at least $f_c/4$mm to sacrifice none of the resolving power of the eye. The objective of a *telescope* should at least be $f_c/4$ mm to realize the full resolving power of the eye. For a *microscope* objective $v_1 = 160$ mm. hence the condition for full·resolving power gives $2r/160 = 4/f_c$ or numerical aperture$=640/f_c f_b$ ($f_b=$ focal length of objective), much greater than any used. Hence microscopes are no strain upon the eye as regards resolution while a telescope with $F/5$ objective would require an ocular of 0.8 mm. focal length to realize the full resolving power of the eye.

In the *photographic plate* the silver grains measure 1 to 6μ across, say 3μ an on average. Images to be resolved must be five to ten times this distance apart, say 20μ on a good plate. Using light of wave length 0.5μ, a photographic objective, if free from aberrations, would require an aperture ratio of but $F/2r = 60$ to realize the possibilities of the plate.

The resolving power of the *eye* (and of any objective of similar aperture ratio $F/5$) corresponds to an angular separation of 0.00015 or 0.15 mm at a distance of one meter, 0.04 mm at 25 cm (10 inches) or 24 cm at one mile. The resolved details of a photographic plate (20μ) at 25 cm are just beyond the full resolving power of the unaided eye.

Residual Aberrations.—The diffuseness of an image due to residual aberrations may of course be eliminated to some extent over a limited field and for objects of certain colors at certain distances. Within the working field this diffuseness must in some cases be brought down to the limit set by the resolving power of the photographic plate. Thus the limits are either dependent on the aperture ratio of the

objective (*visual* systems) or else numerical fixed quantities, in the case of *photographic* systems.

An image pencil coming to a focus at a distance δv from the chief image surface is spread out at that surface over δv *tan* ϕ is the angle of the pencil. With sufficient precision then the spreading of the image is $S=A\delta v/v$, A being the aperture of the image pencil. Putting this equal to the diffusion $\lambda v/A$ due to the limit of resolving power, the permissible departure δv from the image plane is

$$\delta v=\lambda v^2/A^2.$$

Further, since in this case the lateral diffusion (S) in the image is $\lambda v/A$, that diffuseness may be best expressed in terms of wave length (λ), image distance (v) and lens aperture (A) for, other things being equal, it is proportional to the first two of these and inversely proportional to the third.

If any image is to be photographed, the limit of permissible spreading of the image may be put at 2 or 3 times the width of the silver grains, say 5μ. Hence the permissible departure from the image plane

$$\delta v=\frac{v}{A}5\mu$$

a limit difficult of attainment for lenses of wide aperture.

However, in most ordinary work that is not to be enlarged the limit of permissible diffuseness is set neither by the resolving power of the objective nor of a photographic plate, but by the angular size of the most important details. Good seeing requires that the objects to be observed be of about the angular dimension of this type at 10 inches; larger is difficult to cover, smaller is too fine to see with greatest ease. Modern type is the result chiefly of development toward comfortable vision alone and thus, in size and form, represents real physiological data. Call the mean line width 1/10 the height of the letters and the permissible diffusion at the edges of the line 1/10 of this width. Best seeing is then in angle 1 : 125 (2 mm. at 25 cm.) while the

limit of diffusion from the standpoint of visual acuity is 1 : 12500 (0.02 mm. at 25 cm.) hence

$$\delta v = 0.02 \frac{v}{A} mm$$

which may be regarded as a practical limit to safe departures from the image plane whether due to residual aberrations or to defective focussing except in high power astronomical work or photographs to be enlarged.

Testing.

While the mathematical test of an optical system by computation of rays through it is perfectly rigid and discloses the exact nature of the defects, it is extremely laborious and is inapplicable when the exact data (radii, indices of refraction, thicknesses and separation) are unknown, hence experimental optical tests must be resorted to. These often disclose the nature of the defects at a glance and in many cases admit of precise measurement.

Rough Visual Tests.—Residual chromatism and lack of stigmatism (spherical aberration, coma and astigmatism) in a small objective may easily be detected and roughly estimated by merely holding the objective in one hand and a magnifier in the other and viewing the image of a convenient distant object. The magnifier should be of but moderate power, say of 1/5 to 1/3 the focal length of the objective tested. The test object should contain sufficient fine detail and contrast, like the twigs of a tree against the sky. Winter snow scenes are particularly effective in revealing residual chromatism. This method is an old and deserved favorite with experienced workers, and surprisingly efficient to a novice.

Sources and Test Objects.—The stars are, of course, ideal test objects except as regards availability. Laboratory substitutes are the illuminated pin-hole or mercury droplet and the minute image of a distant powerful source (arc or Nernst filament) formed by a very short focus ocular or Abbe micro-

scope condenser. A bare Nernst lamp with separate rheostat control and an arc with projection lantern are indispensable adjuncts of a testing laboratory. Convenient distant test objects are distant landscapes, buildings, preferably with brick walls, sheets of coordinate paper and test charts formed of sets of geometrical figures ruled on Bristol board. The best focal test objects are half-tone screens, gratings of fine wire, the Abbe test plate, and Grayson rulings. Scraps of trimmings of half-tone screens of ample size may be had at slight cost and make admirable test objects. The rulings run from 50 to 200 per inch. The black lines are dense and equal in width to the spaces between them, and the rulings are very precise. The Abbe test plate is of gold-leaf on glass ruled so as to tear and give sharp corners. Grayson rulings are on glass, 1/3 to 1/20 as wide as their separation and unfilled. They are supplied in sets 1000 to 10,000 in steps of 1000; 10,000 to 120,000 in steps of 10,000, etc. They are excellent test objects for microscope objectives.

Spectroscopic tests for chromatism and spherical aberration are easily made with any small spectroscope; a pocket spectroscope does very well. An image of a wire, half-tone screen or artificial star is thrown on the slit of the spectroscope with the lens to be tested, with the lines across the slit. The test object should be illuminated with light that is as white as possible in order to avoid weakness at the blue end of the spectrum. When properly focussed, the spectrum is seen channeled throughout its length by heavy bright and dark bands. With the lens axial, differences of focus will be found for different colors of the spectrum, and with refined methods, residual axial chromatism may be determined as a function of wave length. If spherical aberration is present, the dark bands will have more or less light diffusing into them from the light bands. If this lateral diffusion of light vanishes at two colors, the Gauss condition is satisfied for those wave lengths.

The lens may be tested obliquely for coma and oblique

chromatism by similar methods. Microscope and large telescope objectives are easily given a simple chromatic test by using a spectroscopic ocular or by inserting a direct vision prism before an ordinary ocular.

Determination of Gauss Points and Equivalent Focal Length.—These determinations require a bench of special construction providing means of rotating the lens about a determinable transverse axis. A slide carries the lens holder which is mounted on a second slide and provided with means of rotation about a vertical axis. To determine a Gauss point the lens is moved with respect to the transverse axis until the image of a distant object, viewed with a microscope or high power ocular, is not displaced by a slight rotation of the lens. The transverse axis then passes through the Gauss point corresponding to the image side of the system. By reversing the lens, the other Gauss point may be determined in a similar manner. The equivalent focal length is the distance between the image and the corresponding Gauss point. Determinations of equivalent focal length by the magnification methods give unsatisfactory results. The focal length appears as a periodic function of the magnification, due perhaps to the periodic nature of light and also as a continuous function of magnification due evidently to the displacement of the transverse axis (the real apex of the field pencils) with object distance.

Tests for Curvature, Astigmatism, and Distortion.—Simple photographic determinations of these aberrations may be made with a ruled surface like a sheet of coordinate paper or a brick wall as test object. Distortion is determined with the test object normal to the lens axis by a simple comparison of linear distances at the center and edge of the image. To determine primary and secondary curvatures of field, astigmatism and depth of focus, either the plate or test object is inclined a known angle to the optic axis. A plate of fine grain is necessary for precise work. The chief advantage of the photographic method is that it gives a permanent record.

The two curvatures may be directly measured with the aid of a T-shaped bench on which either the test object and observing microscope or the lens may be displaced normally to the axis. Perhaps the most convenient method is to use a fixed test object with movable lens and throw an image on a distant wall. A wire grating illuminated by a Nernst lamp is a good test object. The primary curvature is determined with the wires normal to the primary plane, secondary curvature with the wires parallel to the primary plane.

The Knife-edge Test for Focus, Zonal Errors and Surfaces.—If a lens or mirror is viewed by an eye placed at a point image (of a star, say) the whole optical surface appears as a blaze of light since the whole image pencil enters the eye pupil. If the eye is withdrawn slightly and a knife edge brought up so as to nearly but not quite cut off the image pencil at the image point, a similar appearance is observed, but the brightness is cut down so that slight inequalities may be observed. If the knife edge is not just at the focus, the illumination of the surface will appear one sided, as is evident from a consideration of the rays cut off. For instance if too near, the side away from the knife edge will appear the brighter. This method of locating the focal point is much more sensitive than the ordinary one of setting for maximum sharpness of image and is particularly adapted to well corrected systems of large relative aperture. According to Ritchey[2] the 60 inch reflector at Mount Wilson (focal length 300 inches) may be thus focused to 0.025 mm. Inequalities of curvature are at once apparent as inequalities of illumination in the knife-edge test. The method is particularly applicable to systems like telescope objectives having few surfaces. Defective color correction is easily detected if the source is white. If monochromatic illumination be used the focal length may be determined for each wave length separately.

The results of a knife-edge test may be easily recorded photographically. The camera must be near enough to

include the whole light cone and focussed on the optical surface tested.

Tests of Surfaces by Interference Methods.—Optical surfaces both plane and spherical, may easily be tested for uniformity by interference methods if another similar surface of opposite curvature is at hand. The pair of surfaces are carefully freed from dust particles and laid together so as to be parallel and almost in contact. Illumination with diffuse light gives interference rings and from the width and symmetry of these rings the perfection of the surfaces is judged. A slight displacement of one surface shows to which of the two surfaces any visible defects belong.

The interference figures are more visible in a darkened room with the optical parts protected in the rear with black velvet or else placed over a hole in a closed blackened box. Ground-glass is a good diffusing screen. The interference is much sharper if monochromatic light be used. A sodium flame is commonly used; tubes of conducting mercury vapor or helium give more brilliant sources.

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III.

OPTICAL INSTRUMENTS.

Optical instruments fall into two distinct classes. One class includes all those instruments used merely to form an image of some object; such instruments as the telescope, microscope, field glass, and photographic objective. It is this general class of instruments to which this chapter is devoted.

A second class of instruments includes all those for special laboratory purposes, used chiefly to analyze light as to its quality or quantity in relation to the properties of some body or material. Photometers, spectrometers, colorimeters, polariscopes and many of the other kinds of instruments are of this class. These are taken up one at a time in the remaining chapters.

The General Properties of Instruments for Forming Images.

All optical instruments that are used chiefly for forming images, either for visual inspection or photographic record, may be classified and rated according to the degree of their attainment of

1. Definition,
2. Magnification,
3. Illumination,
4. Angle of view,

in the image formed. Within limits these should all be as great as possible, but since each of these is in general secured in high degree only by the sacrifice of others, the design of all instruments of this class is a compromise favoring those properties most desirable for the use to which

the instrument is to be put. In some instruments (*e.g.*, opera glass, reading telescope, spectacles, hand magnifier) the requirements are not severe and are easily met. In others the severest demands on the principles of construction are made. Other special properties are desirable in certain instruments and are discussed later.

Definition.—In securing high definition, three classes of limitations are encountered: (1) the *resolving power* of the objective, ocular or eye, (2) *residual aberrations* in parts of the instrument or incomplete compensation of aberrations between these, and (3) the *granular structure* of the retina or photographic plate. These limitations were discussed in some detail in the preceding chapter, and are here merely reviewed in relation to complete instruments.

The resolving power of its objective sets an absolute limit to the definition attainable by any instrument. The resolving power of succeeding parts of the instruments should be sufficient to sacrifice none of the definition secured by the objective. In visual instruments, since the resolving power of the eye is fixed by the pupil aperture and retinal grain, the ocular must on the one hand be of sufficiently short focal length to present to the eye an image within its range of resolution and on the other hand of sufficient aperture to sacrifice none of the detail presented by the objective.

If ϕ is the angular separation of the details (double stars, diatom markings) to be resolved, $\phi = a\ \lambda/r$ where λ is the wave length of the light (0.00005 cm for green) used, r the radius of the objective and a is a constant equal to 0.5 for details barely resolved, or $a = 1.0$ for sharp resolution. If either object or image is near the principle focus of an object, $\phi = \delta/F$ where δ is the linear distance between the details and F is the equivalent focal length of the objective. Hence details to be observed by the eye must subtend an angle of at least half a minute of arc or roughly 0.1 mm at a distance of 1 meter.

The resolving power of an instrument is that of its objective provided the succeeding parts (ocular and eye) have sufficient

aperture to take in the whole of an axial pencil filling the objective.

Lack of definition due to residual aberrations in the components of an instrument may be avoided not only by eliminating aberrations in each component (see last chapter), but by balancing residuals in different components. In some of the best microscopes the ocular removes part of the residual aberrations of the objective.

The third limitation to the securing of high definition, the granular structure of the eye and photographic plate, must be avoided by suitable magnification. These are both of the order of magnitude of 2 to 5μ and sharp definition in an image requires that the details to be observed be five to ten times this, say 20 to 40μ (0.02 to 0.04 mm) on either the retina or photographic plate.

An instrument is of normal definition if both resolving power and correction of aberrations give as high definition as the eye or photographic plate over the field to be covered.

2. **Magnification.**—The size and position of the image produced by an optical instrument of the image forming class is of course of prime importance. The best *size* is determined by the limits of comfortable vision for the details to be observed and may be set roughly as the size of the type at 10 inches, 0.008 in angular measure or about 1 in 125, but details one-tenth or ten times this size may be observed without strain.

The *position* of the image may vary within wide limits, in visual instruments. The normal human eye can accommodate itself to rays diverging from a point as near as 25 cm in front of the eye, parallel, or converging to a point a meter behind the eye. In focusing a visual instrument, we form the image anywhere from about 20 cm to an infinite distance in front of the eye. There is little to choose between divergent and parallel light, so long as the angular size of the image is not greatly altered.

In photography, if the plate or print is to be viewed directly, the details to be observed should be in size about 2 mm for

comfortable vision. In photographing distant objects, the linear size of image is the angular size of the object times the focal length of the objective.

There are several kinds of magnification to be distinguished in its exact formulation. What we may call simple or *screen magnification* is simply the relative linear dimensions of the same detail of image and object. *Visual magnification* is the relative angular size of a detail of image and object as viewed from some arbitrary eye point on the axis. Again visual magnification may be either absolute or accommodated, depending on whether the image is in the plane of the object, at a distance of 25 cm from the eye or at infinity.

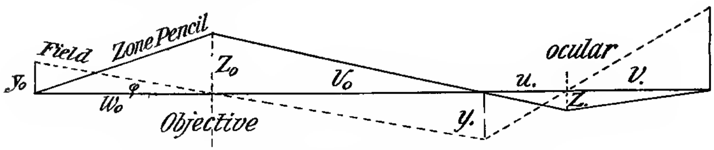

FIG. 24.—Magnification, field and zone pencils, entrance and exit pupils and windows.

In the figure a field pencil (angle ϕ) and a zone pencil (angle z) are shown traversing a two lens optical system, the lenses being represented by traces L_1 and L_2 through their transverse axes. We have for the

$$\text{Linear magnification } m_i = \frac{y_1}{y_0} = \frac{v_0}{u_0} \cdot \frac{v_1}{u_1} = \frac{z_0}{z_1}$$

$$\text{Angular magnification } m_a = \frac{\phi_1}{\phi_0} = \frac{v_0}{u_1} = \frac{v_0 z_0}{u_1 z_1}$$

Hence the linear magnification of a system is the product of the linear magnifications (v/u) of its separate components. Or it may be taken as the relative vergency (z_0/z_1) of initial and final *zone* pencils. Angular magnification is the relative vergency (ϕ_1/ϕ_0) of initial and final *field* pencils or the relative diameter $(v_0 z_0/u_1 z_1)$ of entrance and exit pupils.

The linear magnification is ordinarily used for computing the magnification of microscopes (since the image lies nearly

in the object plane), angular magnification for telescopes. For microscopes v_0 = tube length = 160 mm, v_1 = image distance = 250 mm, u_0 = focal length of objective (roughly) and u_1 of the ocular, hence $m_i = v_0 v_1 / u_0 u_1 = 40{,}000$ mm divided by the products of the focal lengths of objective and ocular (in mm). For telescopic systems $v_0 = f_0$ and $u_1 = f_1$ hence the angular magnification is simply f_0/f_1.

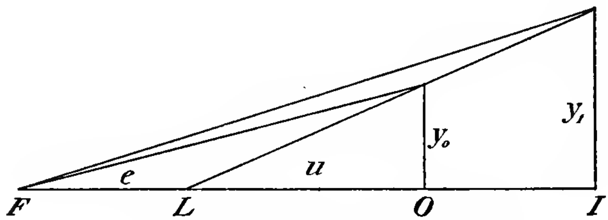

FIG. 25.—Magnification, simple magnifier.

Consider the simple case of a single lens interposed between the eye and the object. Image (y_1) and object (y_0) will subtend the same angle at the lens, but to an eye at a distance e from the lens, the relative angular size of image and object is

$$y_1/(e+v) : y_0/(e+u) \quad or \quad (e/u + 1)/(e/v + 1)$$

The distance e is measured from the inner nodal point of the eye, u, v and e from the transverse axis of the lens.

In the general case for a single lens then

$$m = \frac{\phi}{\phi_0} = \frac{e/u + 1}{e/u + 1} = \frac{e\left(\frac{1}{f} - \frac{1}{v}\right) + 1}{e/v + 1}$$

is the complete expression for the magnification. This would hold for a reading glass. In a high power magnifier, the image distance v is large in comparison with eye distance e and focal length f and $m = e/f + 1$, or simply e/f if f is small compared with e. These formulas hold for compound as well as simple lenses. For systems consisting of objective

and ocular the modification for eye distance may easily be introduced in the special cases.

Normal magnification in any visual instrument is such as gives an exit pencil just filling the pupil of the observer's eye. Higher magnifications sacrifice both illumination of image and field angle without any gain in resolving power, but are used in some cases where the illumination, field and definition are ample and large images desirable.

3. **Illumination of image** is of great importance in all optical systems, particularly in those of high power and for photographic purposes. The *quality* of the illumination, its *direction*, its *intensity*, variations in intensity producing *contrast* and the *falling off* in intensity from the center of the field outward have all to be considered.

An object viewed through any optical system appears of the same intrinsic brightness (aside from losses by absorption and reflection) as when viewed through any other system or viewed directly by the eye, provided the pencil of light entering the eye is as large or larger than the pupil. In other words, within these limitations, the brightness of the image does not depend upon the instrument used, its magnification, nor upon the size and location of the image. The apparent brightness is proportional to the area of the eye pupil when this is filled by the image pencil.

When the image pencil is too small to fill the pupil, the brightness of the image falls off in the ratio of the area of the pencil to the area of the pupil opening. Since the radius of the exit pupil is VN/m for the microscope and r/m for the telescope (V = visual distance, 25 cm, $N = r_1/u_1$, r = radius of objective, m = magnification and p radius of the eye pupil), we have for the brightness relative to the normal brightness

$$B : B_0 = (VN/m)^2/p^2 \text{ or } r^2/m^2 : p^2$$

$B = B_0$ gives the value of the normal magnification mentioned above.

When the object is too small (*e.g.*, a star) to be resolved

by the eye or optical system, the light reaching the image is spread out over a fixed area. Let A be the aperture ratio of the objective and λ be the wave length of the light used and for visual instruments assume that the ocular is such as to bring the image within the resolving power of the eye. Then the limiting diffusion is $\delta = \lambda A$. The brightness of the circular disk·image of a point object is inversely proportional to δ^2 or to $A^2\lambda^2$. The brightness of the band image of a fine line source is inversely proportional to δ, hence to λA.

When an image is projected on a screen or photographic plate the brightness of image depends upon the total light falling on unit area instead of the light within a given cone as with visual instruments. The relative illumination (light per unit area) of image and object is

$$\frac{I_1}{I_0} = \frac{TS}{v^2}$$

where S is the area of the free aperture of the objective v image distance and $1 - T$ is the percentage loss of light by absorption and reflection.

When the object is a very fine point, the relative illumination of diffraction disk and objective is inversely as their areas

$$\frac{I_1}{I_0} = \frac{D^2}{\delta^2} = \frac{D^4}{\lambda^2 F^2}$$

since $\delta = \lambda F/D$. Similarly when the object is a fine line

$$\frac{I_1}{I_0} = \frac{D^2}{\delta} = \frac{D^3}{\lambda F}$$

Contrast in the image is of course the same as in the object except as regards fine details beyond the limit of resolution of the objective. Bright details on a darker background (*e.g.*, stars in the sky) gain in contrast, while darker details on a light ground (canals on Mars) lose in contrast by increasing the aperture of the objective. De-

fining contrast as the relative brightness of field and detail then the relative contrast of image and object is proportional to F/D^2.

Self luminous objects of course completely fill the objective cone with light. The illumination of *non luminous* objects should be such as to fill the objective cone, otherwise both resolving power and image illumination will of course be reduced as though the aperture of the objective had been correspondingly reduced.

4. **Angle of View.**—The beam of light passing through an optical system is limited on entering by the entrance pupil, on leaving the exit pupil, defined by the initial and final apertures of the axial zone pencil. The angle of view is the maximum angle of the field pencils of rays, the field pencil on the object side is limited by the entrance window and on the image side by the exit window. But the entrance window is the image of the exit pupil formed by the whole optical system and the exit window is the image of the entrance pupil. The angular field covered by an optical system is the angle subtended by the entrance window at the entrance pupil. In visual systems, the angle of view is similarly the angle subtended by the exit window at the exit pupil, it is the field angle multiplied by the magnification (see Fig. 24).

When the exit zone pencil is as large or larger in aperture than the eye pupil, the image of the eye pupil is the entrance window limiting the field of view. Such a system is used most efficiently, therefore, when the eye pupil coincides with the exit window of the system.

Field and visual angles may be easily expressed in terms of apertures and distances for any optical system. For example consider a system consisting of objective and ocular. Let f_0 be the equivalent focal length of the objective, u_0 be the object distance, and D_0 the diameter of the entrance pupil, roughly the diameter of the objective, more precisely the circle on which edge rays of the axial zone pencils intersect. If s is the separation of the objective and ocular

(between transverse axes) then the diameter of the exit pupil is

$$D_1 = D_0\left(\frac{s}{f_0} - \frac{s}{u_0} - 1\right)$$

provided lens rims in the ocular do not cut it down. The field angle is

$$\phi_0 = D_0\left(\frac{1}{f_0} - \frac{1}{u_1} - \frac{1}{s}\right)$$

while the visual angle is simply $\phi_1 = D_0/S$. The diameter of the exit window $D'' = D_0 f_0/(s-f_1)$ and its distance from the exit pupil $v'' = sf_1/(s-f_1)$, f_1 being the equivalent focal length of the ocular. The separation s, of course, depends upon the distance v of the image for which the observer focusses the instrument. It is

$$s = \frac{u_0 f_0}{u_0 - f_0} + \frac{v f_1}{v - f_0}$$

Photographic objectives have only an entrance pupil and have no exit pupil, entrance window, nor exit window except when the lens mounting is narrow or long. Since the field pencils fall off in definition at smaller angles of obliquity than the angles at which the illumination falls off, the size of plate covered is determined by the definition rather than by the illumination.

In certain recent terrestrial (so-called wide angle-) telescopes, the ocular is made of very wide aperture so that the eye may be moved about to view different parts of the image. The actual field angle is not increased, but the entrance pupil is in effect movable, the net result being that a linear motion of the eye may be substituted for an angular displacement of the instrument.

The Image Forming Instruments.

Photographic Objectives.—In photographic objectives, whether for astrophotography, general camera work, or for copying and enlarging in the laboratory, four qualities are

of paramount importance: (1) *Definition*, (2) freedom from *distortion*, (3) *illumination* ("speed"), and (4) the *field* covered ("covering power") on a flat plate; in other words, both field and zone pencils should be stigmatic and as large as possible. These properties have already been discussed separately. In practice, field and definition are sacrificed to secure high speed and speed is sacrificed to secure a wide field.

Speed.—The relative illumination of image and object is $\pi\ sin^2\ V$ where V is the angle between extreme edge rays coming to a point in the image. Speed is therefore proportional to $sin^2\ V$, the square of the numerical aperture of the lens. An $f/8$ lens is four times the speed, and will take a photograph in one-fourth the time required by an $f/16$ lens.

In the natural octave system of stop openings the apertures are proportional to

f : 1, 1.4, 2, 2.4, 4, 5.6, 8, 11, 16, 22, 32, 45, 64, 91, 128.

The stop numbers adopted by the Paris Congress are:

No. stop	1/6	1/5	1/4	1/2	2/3	1	2	4	8	16	32	64
Rapidity ...f:	4	4.5	5	7	8	10	14	20	28	40	56	80

The Steinheil stop numbers are:

f : 4.5, 6.3, 7.7, 9, 12.5, 18.0, 25, 36, 50, 71

while the Zeiss stop numbers are:

f : 3.5, 4.5, 5.6, 8, 11.3, 16, 23, 32, 45, 64, 91, 128.

Each stop system has its disadvantages, being either too artificial or else having too few intervals in the range from $f/4$ to $f/8$ most used. Writing

$$\frac{I_1}{I_0} = \pi\ sin^2 = V\frac{\pi r^2}{f^2 + r^2}$$

for the relative illumination of image and object and neglecting losses by absorption and reflection in the lens itself, we have for

f/D =	0.37	0.74	1.0	1.10	1.70	2.46	2.76	5.0	8.0	10.0
I_1/I_0 =	2.0	1.0	0.63	0.50	0.25	0.125	0.10	0.031	0.012	0.0078

The effective *stop opening* is the diameter of a zone pencil entering the lens coming from an axial point object at

infinity. It may be measured by placing a point source at either principal focus and measuring the diameter of the circle illuminated on a ground-glass screen placed over the opposite face of the lens.

Losses by absorption and reflection amount to from 20 to 50 percent in ordinary corrected objectives. The loss by reflection amounts to roughly 10 per cent. for each pair of free surfaces. On cemented surfaces the loss by reflection is so slight as to be negligible. Losses by absorption are seldom over 5 per cent., but a lens containing glass visibly yellowish in tint may absorb as high as 25 or even 50 per cent. in the blue and violet.

Diffuse light and flare in the image are caused by light twice reflected within the objective by its free surfaces. About 1/400 of the light is thus doubly reflected by each pair of free surfaces in the direction of the image. This light is of course in general not in focus at the image plane and produces merely a slight and negligible reduction in contrast, unless a brilliant object like the sun is shining directly on the lens or unless a large light background is in the field. When two consecutive free surfaces in the objective are nearly parallel, the diffuse light is nearly in focus in the focal plane and the image contains a "flare spot."

Photographic objectives are called high speed, large zone or wide aperture objectives if from $f/2$ to $f/5$, medium speed from $f/5$ to $f/8$, slow from $f/8$ on. Wide angle objectives run from $f/12$ to $f/20$ and smaller.

High Speed Objectives.—The highest speed objectives ($f/1.8$, $f/2$, $f/3$) are used for taking and for projecting photographs for moving pictures, and to some extent in astrophotography for faint sources like comets' tails and nebulæ. They cover from 10 to 20 degrees with fair definition. Slower objectives ($f/4$ to $f/5$) covering 20 to 50 degrees are used for portraits and snap-shot photography. The 60-in. reflector at Mount Wilson is $f/5$ when used photographically, the plate used being 3 1/2 in. square, a field of but half a degree.

High speed objectives as a class do not differ radically in design from the medium speed objectives from which they are generally derived. The separate lenses are given a larger diameter and the surface curvatures slightly modified to secure the best distribution of the large residual aberrations.

The medium speed objectives, $f/5$ to $f/8$ have received most attention from lens designers and inventors and represent the highest development of lens design. These apertures corre-

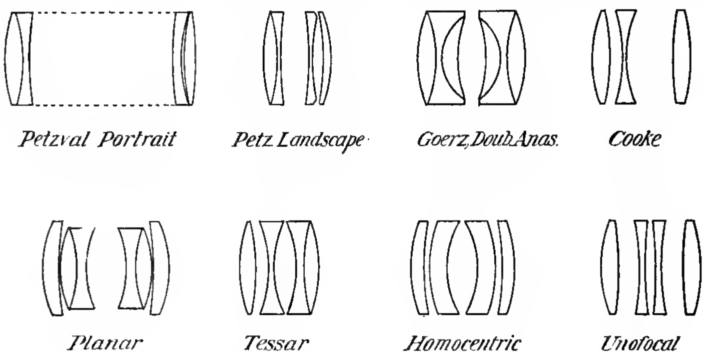

FIG. 26.—Some types of photographic objectives. Front to left.

spond roughly with exposures of from 1 to 10 seconds in-doors and from 1/100 to 1 second in full sunshine. The field covered is usually from 50 degrees to 80 degrees. This class represents the higher grade portrait, group and landscape objectives, objectives for copying and enlarging and the higher grade astrophotographic refractors.

Outline sketches of sections of various well-known objectives are given in the figure. The exact numerical data are not given, for these are as a rule known only to the manufacturers and are constantly being modified in accordance with slight differences in glass meltings. The data given in patent specifications often differs considerably from that finally used.

Wide angle objectives range from $f/12$ to $f/20$ covering from 90 degrees to 110 degrees. In their design attention is given

to lengthening out the oblique pencils to secure a wide, flat field free from distortion, hence they differ from a wider aperture lens merely stopped down. They are used chiefly for obtaining true perspective in architectural photography and to some extent in copying, since a smaller, less expensive lens may be used instead of a large long focus of medium aperture. For extended groups and landscapes a panoramic camera is used with an ordinary medium speed lens.

Each manufacturer usually puts out from three to five series of objectives of one or more types, and of 5 to 15 focal lengths in each series for plates of various sizes. To these ordinary series (say $f/2.5, f/4.5, f/6.3, f/8, f/12$) is frequently added a series of about $f/9$ for three color work, carefully freed from all chromatism at *three* different wave lengths and a telephoto objective for obtaining large images of distant objects.

Telephoto objectives.—In photographing a distant object, the size of the image on the plate is the angular size of the object multiplied by the focal length of the objective, hence to secure large pictures of distant objects an objective of large focal length must be employed. An ordinary long focus objective is objectionable on account of its inconvenience and unsteadiness. Since focal length is measured from image to Gauss point, these disadvantages may be avoided by throwing the Gauss point out to a suitable distance in front of the objective and keeping the objective at a suitable distance from the plate.

The Gauss point is thrown forward and the focal length increased by inserting a negative lens behind an ordinary objective. The required combination of focal lengths may readily be determined by the first order theory and the system then freed from aberrations by general methods. The field required is small.

Microphotographic objectives are simply smaller sizes (20 mm) of the medium speed ($f/4.5$ to $f/8$) objectives. Ordinary microscope objectives are corrected for a fixed image distance (160 mm) and do not give a well corrected

image at greater distances. For extreme resolution and magnification, the microscope objective is retained, but the ocular replaced by a small ordinary objective.

Magnifiers and Oculars.—These are used for viewing objects or real images at close range and producing more or less magnification. Their properties are intimately related to the properties of the eye. Since in vision (1) attention is fixed on but a small portion of the image at a time, (2) the visual accommodation is considerable, (3) the retina is relatively insensitive to all but a few wave lengths in the yellow green and (4) the resolving power of the eye and retina are limited, hand magnifiers and oculars do not require the high corrections given objectives. The three aberrations given most attention are distortion, curvature and oblique chromatism.

Hand magnifiers have been given a wide variety of forms ranging from a single lens (reading glass) to a complete

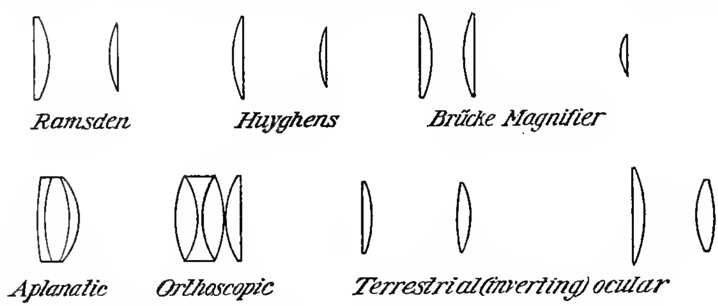

FIG. 27.—Some types of oculars and magnifiers.

microscope (Brücke magnifier). Oculars are chiefly of two standard types, the Ramsden and Huyghens, although many forms of compound cemented systems have been developed. A few of the best known forms are outlined in Fig. 27.

The Huyghens ocular consists of two separated plano-convex lenses with their convex sides facing the objective. The separation is made half the sum of their focal lengths to satisfy the condition for axial achromatism. The focal

length of the field lens is usually three times that of the eye lens, but for low magnifications a smaller ratio is used. This ocular forms an image at the visual distance (25 cm to ∞) from the eye when the image formed by the objective falls between the two lenses of the ocular hence it cannot be used conveniently with an ocular micrometer. The compensating ocular used with high power apochromatic microscope objectives is a related form specially corrected to neutralize oblique chromatic differences due to the objective.

The Ramsden ocular consists also of two separated planoconvex lenses, but with convex sides turned toward each other. The two lenses are of equal focal length and the condition of achromatism would therefore require a separation equal to this focal length. But this separation would give the undesirable condition of field lens being in the focus of the eye lens (dust particles on the field lens would be in focus with the image), hence the separation is made shorter than this focal length and the resulting chromatic differences eliminated by making the lenses compound. The Ramsden ocular may be used with ocular micrometers since the image point is in front of the field lens.

Certain oculars are especially designed to be free from reflected images, to have wide flat field, high central definition, careful color correction, and the like with reference to the objectives with which they are to be used.

The eye with the ocular, magnifier or spectacle lens constitute a positive optical system forming a real image and comparable with a photographic objective. The residual aberrations of the system depend upon those of the eye (see The Eye and Vision) as well as those of the ocular itself. The entrance window, limiting the field pencils, is the image of the eye pupil formed in front of the ocular by the ocular. The entrance pupil limiting the zone pencils and hence determining the brightness of the retinal image, may be an image of either a lens rim in the ocular or of the eye pupil itself. The oblique aberrations are of considerable consequence in the ocular, but of slight consequence in the eye,

on account of the rotation of the eye, the use of but the axial portion in direct vision and the natural curvature of the retina.

Telescopes.—The requirements to be met by the objectives of the many forms of telescopes are very similar, but are very different from those to be met by the photographic objectives. In telescopes the field to be covered is very limited, from 1/4 degree to a few degrees at most, but within that field the definition must be nearly or quite perfect, that is, as high as the resolving power of the objective will permit. Astigmatism, curvature of field and distortion are ignored (except in some photographic telescopes) attention being centered on the two axial aberrations, spherical aberrations, and axial chromatism, and upon the aberrations varying as the first power of the distance from the axis, namely coma and oblique chromatism. In objectives of large aperture ratio ($f/10$ or over) the chromatic difference of spherical aberration (Gauss condition) and the fifth order spherical aberration must be removed.

The chromatic differences eliminated are for photographic telescopes between the violet mercury line 404 and the blue hydrogen line 486; for visual telescopes, the rays brought to a common focus are in the green and yellow at wave lengths about 550 and 600$\mu\mu$, on either side of the maximum of luminosity of most highly incandescent bodies (see Chapter V). Secondary spectrum is eliminated by careful selection of glass. Reflecting telescopes are of course entirely free from chromatic differences, spherical aberration is eliminated by parabolizing, but coma is not eliminated and is very troublesome at 0.2 degree from the axis.

A few of the leading types of telescope objectives are outlined in Fig. 28. The *Fraunhofer* type is a cemented doublet freed from spherical aberration, coma, and axial chromatism. The *Gauss* type is a separated doublet more carefully freed from chromatic differences than the Fraunhofer. The *Clark* type so much in use is also a separated doublet, the separation being adjusted to eliminate lateral chromatism.

The uncemented doublets and triplets in contact are much used in reading telescopes. The Cooke triplet, devised by Dennis Taylor for astrophotography has not only high central definition, but considerable speed and field as well.

A few of the various telescopic systems are listed below. The simple refractor with positive ocular and inverted image is by far the most used both for astronomical and reading telescopes. In the *equatorial* form of mounting the tube is

FIG. 28.—Types of telescope objectives.

swung on an axis (the *polar axis*) parallel to the earth's axis. In the telescope *coudé* the polar axis is hollow and a mirror reflects the image pencils down it to the observer or photographic plate. The *Gallilean* telescope has a negative instead of a positive ocular, thus giving an erect image and a bright but narrow field. *Terrestrial* telescopes are provided with either negative oculars or with reflecting (Porro) prisms to give erect images. A positive erecting ocular is often used. Reflecting telescopes are arranged in several ways. In the *Newtonian* form a plane mirror at 45 degrees with the axis just inside the principal focus reflects the image pencil to a focal plane just outside the incident pencil and parallel with the axis. In *Cassegrain's* form, the image pencil is doubled back on itself by an axial hyperbolic mirror placed nearly half way from the principal mirror to its principal focus. The reflected beam is reflected outside the incident beam by a 45 degree plane mirror just in front of the main mirror. In the Cassegrain-Coudé this last reflection sends the image pencil down through the polar axis.

The *practical requirements* of telescopes are of several classes. By far the most general requirement for visual telescopes, whether for observing a double star or for reading a distant scale, is that the range of confusion (due to the limit of resolution) be as small as possible in comparison

with size of image. When this has been attained in the objective, the image is eye-pieced up with a suitable ocular to the desired magnification or until the diffusion in the image becomes apparent, since the size of image is ϕf, ϕ being the angular size of object and F focal length of objective and the range of confusion $\delta = \lambda F/D$. The ratio

$$\frac{\phi F}{\delta} = \frac{D\phi}{\lambda}$$

is to be as great as possible, hence D the diameter of the objective is to be as great as possible.

The further requirements of maximum illumination of image and maximum contrast in the image have been sufficiently discussed above under "Illumination."

Binoculars.—Field and marine glasses, and opera glasses are simply low power erecting telescopes mounted in pairs. Field glasses range in magnification from 4 to 10 or 12, higher powers not being much used on account of the difficutly in holding them in position with sufficient steadiness. "Night" glasses are of lower power than day glasses. Opera glasses are usually of about 2 1/2 power, frequently less than this. The limit of the diameter of the objectives is of course the distance between the eye axes, usually taken as 62 mm. or 2 1/2 inches.

The objectives are usually simply cemented doublets or triplets as are the eye-pieces. In the older Gallilean type the ocular is negative and placed inside the focus, thus securing a great saving in length of instrument, an erect image, and a maximum illumination, but a narrow field. In the newer prism binoculars a positive ocular is used and the necessary erection of image and shortening of instrument secured by doubly reflecting (Porro) prisms.

Since only low powers are used in binoculars, excessive definition or correction is not required, and illumination of image and angle of view are chiefly to be considered. The older Gallilean type has but the one serious defect of a very narrow field (seldom over 6° or 1 : 10), but that is an

important one in locating objects. The earlier prism binoculars secured a much wider field, but introduced another defect nearly as serious. To save weight and length the reflecting prisms were made small and placed far apart so that the effective opening of the objectives was reduced from 50 mm to 15 mm. This made the exit pencil so much smaller than the eye pupil that there was a very serious loss of illumination of image. In the recent prism binoculars the objective is made larger (30 mm) and the pair of reflecting prisms placed near together near the ocular.

The use of good binoculars should produce no *eye strain* whatever. Lack of easy seeing is usually due to one or both of two defects; either serious residual *aberrations* in objective or oculars or else lack of parallelism of the axes. The lack of proper adjustment of the distance between axes is serious, but does not produce as great a strain as lack of parallelism. To minimize eye strain due to faulty focussing, instruments with negative oculars should be focussed with an inward motion of the ocular, those with positive oculars with an outward movement.

Aside from magnification of image, the use of binoculars may have two decided advantages over unaided vision. The *penetration* effect is very noticeable in viewing an object through a mass of discrete obstructions like underbrush, a wire screen, a snow storm or even a fog. More field rays can get by the obstructions to reach a large objective than would reach the eye pupil directly. The *stereoscopic effect* is much enhanced by certain types of binoculars in which the distance between the axes of the objectives is greater than between the eye axes.

Microscopes.—The modern microscope for ordinary work has attained a high and probably nearly complete development. In all good microscopes the definition of image is fully up to the theoretical resolving power of the objective, this resolving power corresponding to the numerical aperture.

In Fig. 29 are shown the structure of an ordinary microscope and the paths of rays through the optical system.

Two rays of a zone pencil and two rays of a field pencil are shown passing the mirror and traversing in turn the con-

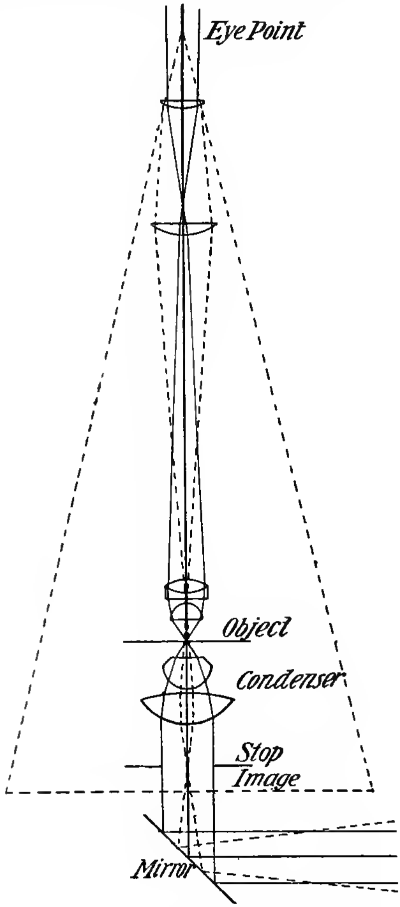

FIG. 29.—Optical diagram of microscope zone pencil in full, field pencil in dotted lines.

denser stop, the condenser, the slide (object), the objective, the ocular and finally forming the initial image somewhere between 20 cm and ∞. The zone pencils converge at the

object and image points; the field pencil converges at the stop and pupil points.

The most effective part of a microscope is of course the objective and the high efficiency of the modern microscope is due to the development of objectives of large aperture and good zonal and chromatic corrections. Objectives range in focal length from 1.5 mm to 50 mm and in numerical aperture from 1.4 down to 0.08. Objectives of 20 to 50 mm focal length (N.A. 0.1 to 0.2) are usually simple cemented doublets; those of 10 to 20 mm focal length (N.A. 0.2 to 0.4) consist usually of a pair of separated cemented doublets. The higher power objectives (10 mm to 1.5 mm N.A. 0.5 to 1.4) are of various forms, but consist usually of two or three cemented doublets or triplets separated by narrow air spaces and with a single hemispherical front lens. *Fluorite* is used for one or two components in some high power objectives in order to secure somewhat higher spherical and chromatic corrections. The *apochromats* developed by Abbe are corrected for three axial colors over all zones, some residual lateral chromatism being taken out by a specially designed ocular.

The shortest focal lengths and largest numerical apertures (hence the highest magnifications and resolving powers) are reached with the *oil immersion* objectives. Cedar oil ($n = 1.51$) placed between objective and cover-glass makes optical contact with the object, bringing into the objective more oblique rays of the zone pencil and eliminating cover-glass corrections. A longer focus water immersion system is useful in observing objects in suspension in water.

All higher power objectives are of course corrected for a particular thickness of *cover-glass* (except those for homogeneous immersion) and for a particular distance of image (tube length) of 160 mm both of which must be adhered to for the best results. Dry objectives are usually corrected for cover-glasses 0.15 to 0.18 mm thick. Less thickness of cover-glass may be compensated to some extent by greater tube length and vice versa.

The *working distance*, that from cover-glass to objective, is of great importance in many kinds of work. An objective of lower numerical aperture is often preferable in securing a correspondingly greater working distance.

The *oculars* used ordinarily with microscopes are of the simple Huyghens type. The compensating oculars used with the apochromatic objectives are either of modified Huyghens or orthostigmat type.

The *condenser* used to illuminate the object observed is of great importance in high power work since the direction as well as the intensity of the illuminating rays are effective in determining the image of a non-luminous object. An objective of wider aperture can be of no service unless (as was shown by Abbe) the illumination is of equally wide aperture. The illuminating rays must approach the object in the same manner that they are received from it by the objective.

The general optical properties of the microscope have been discussed with those of other optical systems. Certain special forms of microscope may be mentioned here. Other forms involving the polariscopic, spectroscopic, or photometric analysis of light belong to later chapters.

Reading microscopes are used extensively for determining the precise location of rulings on metal or glass or of images on photographic plates. Comparatively low powers are used (20 to 50 diameters) and objectives of 20 to 50 mm focal length. Precision of reading depends chiefly upon the quality of the mark observed, the illumination, and to a considerable extent upon the resolving power of the objective.

Reading on good lines should not vary more than 0.02μ from the mean. Readings on circular dots should be made to about 0.01 of their diameter and the same precision is attained with well defined images on photographic negatives. Line rulings are observed with most precision with a bifilar ocular and with magnifications such that the line is from 2/3 to 4/5 as wide as the two threads. Measurements on photographic negatives are best made with a single ocular thread.

While high magnifications are not required or even desirable in reading microscopes, the importance of high resolving power and freedom from aberrations must be emphasized. One has but to stop down an objective to realize how important the resolving power and general quality of image is in precise measurements.

The vertical illumination required for opaque objects may be supplied either by a transparent mirror in the body of the microscope tube, a reflecting prism at the objective, or by a perforated mirror before or around the objective. If the reflecting prism be used its reflecting face should be normal to the line to be observed in order not to cut down the effective resolving power of the objective.

The *Ultra-violet Microscope* is designed to secure high resolving power by the use of very short light waves rather than by high numerical aperture of objective. Since resolving power, other things being equal, is inversely proportional to wave length, ultra violet light of wave length $250\mu\mu$ will give three times the resolving power possible with red light of wave length $750\mu\mu$.

Since glass becomes opaque at about $300\mu\mu$, the objective of an ultra violet microscope to utilize shorter waves than these must be constructed of quartz and rock salt or fluorite which are transparent as far out as (to $180\mu\mu$) air is transparent. Good isolated arc lines for use with the ultra-violet microscope are tellurium $238\mu\mu$ and graphite $248\mu\mu$. The magnesium spark lines at $285\mu\mu$ are commonly used.

The Ultra Microscope.—While the ordinary microscope with N. A. = 1.4 is able to distinctly resolve objects separated by only about the length of a light wave (1/60,000 of an inch or 1/2000 of a mm) objects very much smaller may be seen by diffracted light. The ultra microscope is arranged especially for observing such very minute particles, colloidal particles, Brownian movements, minute organisms and the like. It is simply an ordinary microscope of medium power (4 mm to 8 mm objective) but with the object illuminated only by rays too oblique to enter the objective. Thus the

particles appear bright (by diffracted light) on a dark background. An ordinary Abbe condenser serves fairly well if provided with a large central stop so that only edge rays enter. The slide upon which the objects are mounted is laid directly on the top face of the condenser with cedar oil or water making optical contact between. An abundance of light is required, preferably that from an arc projection lantern.

Special forms of condenser (paraboloid, cardioid, etc.) have been devised to give more oblique illumination of higher intensity and a darker background. With the ultra microscope, particles ranging in diameter from the limit of distinct resolution ($500\mu\mu$) down to about $2\mu\mu$ have been observed. Molecular diameters are of the order of 0.1 to $1.0\mu\mu$ (10^{-8} to 10^{-7} cm.).

Projection Systems.—In projection systems the limit of permissible diffusion in the image is that perceptible to the eye, in angle about 1 to 5 in 10,000, and the definition given by the projection lens should be above this limit. The flatness of field and freedom from distortion should be about the same as with a good ordinary photographic objective. The field covered is moderate, about 1/3 or 20 degrees in angle. The apertures of the zone pencils used vary greatly in different systems and in any one system varies with the form of condenser used, and with the position of the illuminating source with respect to the condenser. If the condenser forms an image of an arc source at the projection lens, the effective aperture of the lens may be much less than its true aperture and the definition correspondingly reduced.

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IV.

REFRACTOMETRY.

The refractive indices of various common substances of fixed composition such as water, hydrogen, air, the mineral crystals, various chemical compounds and the like, have been determined with considerable precision and recorded. Work in refractometry lies chiefly along five different lines, (1) redetermination of known refractive indices with increased precision (2) extending these determinations further into the ultra-violet and infra-red and into absorption bands, (3) identification of unknown substances by means of their refractive indices, (4) investigations of variations in indices of optical glasses, each melting of which has new optical constants upon which its use depends.

Determinations of the refractive indices of substances may be made by several different methods, the choice of method depending upon (1) the precision desired, (2) the nature of the material, whether solid, liquid or gas, and (3) if a solid, on the optical surfaces available. A few of the most useful methods are outlined in this chapter. Methods applicable to metals and dense dyes, in which both absorptive and refractive indices are deduced from the change of phase and rotation on reflection, are discussed in the chapter on Polarimetric Analysis. Other special methods are mentioned under Radiometry and Interferometry. Most direct methods are goniometric, involving the measurement of either a single refraction angle or an angle of total reflection. Differential (comparative) methods involve either the measurement of an angle or the displacement of interference bands. Goniometric measurements are made with a spectrometer, inter-

ference measurements with some form of interferometer. The so-called refractometers are of either the spectrometer or interferometer type, with scales either angular or linear, or empirically graduated to read refractive indices directly.

Simple Refraction Methods.

The method most commonly used in the determination of refractive indices is the most general in its application and also the one capable of the highest precision; namely, the method of measuring the deviation in a ray of light caused by a prism of the substance investigated. If the substance is a transparent solid, a prism is cut from it, if a liquid or gas, a hollow prism with plane parallel glass faces is filled with it. With a good prism and spectrometer, the fifth decimal of the index is easily attainable, and with care the sixth place, while with a specially designed spectrometer, temperature control and other precautions the uncertainty may be reduced to one or two units in the seventh place. The inhomogenuity in ordinary optical glass, impurities in natural crystals, and uncertainties in various corrections frequently affect the sixth decimal place.

Consider a prism bounded by two planes separated by an angle p traversed in a plane normal to both bounding planes by a ray of light. Let $i_1\, r_1$ be the first angles of incidence and refraction, $i_2 r_2$ the second. Then the fundamental relations are

$$\begin{aligned} n_a \sin i_1 &= n \sin r_1 \\ i_2 &= p - r_1 \\ n \sin i_2 &= n_a \sin r_2 \end{aligned}$$

n being the index sought and n_a that of the ambient air. In practice i_1 p and r_2 are determined with the spectrometer, and n/n_a computed from them.

In precise spectrometry i_1 is commonly made equal to r_2 by rotating the prism into that position for which the total deviation is a minimum, a position readily found. In

this case the refractive index is given by the simple relation

$$\frac{n}{n_a} = \frac{\sin\,(\triangle + p)/2}{\sin\,p/2}$$

$\triangle$ being the minimum deviation.

The spectrometer used to determine prism angles and deviations consists essentially of two telescopes directed toward a common axis about which one of them is free to rotate. The other (the collimator) is usually fixed rigidly to the base and is fitted with a slit instead of an ocular. A central table fitted with leveling screws carries the prism. Angles are read on a graduated circle. The prism table may be made to move either with or independently of the circle by means of a clamp screw.

The three chief steps in the use of the spectrometer are (1) adjustment of the prism faces normal to the axis of the telescope, (2) determination of the prism angle and (3) determination of the minimum deviation. The first two are made by using the Gauss ocular or some similar autocollimation device, and thus setting the axis of the telescope normal to each face of the prism in succession.

The *precision* attainable in determining n depends partly on the sensibility of the spectrometer and partly on errors in the instrument or in its use. The *sensibility* may be varied in either of two ways. First the prism angle may be increased until the rays enter and leave the prism nearly parallel to the surface. This involves a prism angle of 70 to 80 degrees. It gives a large variation of deviation with index, but increases errors due to lack of planeness in the prism faces and in practice prisms of about 60 degrees are used. Second, the settings may be made sensitive by (a) the use of occulting wire instead of cross hairs, (b) high power ocular and (c) great length of telescope; when all three are pushed to the limit, the full resolving power of the telescope is utilized. The illumination must be adjusted to a moderate intensity sufficient for good seeing but not so great as to produce sensible retinal irradiation. With a

good ordinary laboratory spectrometer using cross hairs, settings may be repeated to about 10″ of arc. If the cross hairs are replaced by an occulting wire covering about 4/5 of the illuminated slit, the error of setting is reduced to about 1″ while a good precision instrument with occulting system and high power ocular should give readings to 0.1 or 0.2″, a limit approaching astronomical precision.

The *sources of errors* in spectrometry are very numerous but with good objectives only a few are of the first order. The formula shows that an error in measuring the *prism angle* p gives an error in the index of $dn = 0.7\ dp$ approximately, hence an error of 1′ of arc in p means an error of 0.0002 in n, 1″ corresponds to 0.000004 in n. Similarly with a 60-degree prism and a deviation of 44 degrees corresponding to an index about 1.58, $dn = 0.6\triangle$, hence an error in measuring the *deviation* produces an error in n of about the same amount as a similar error in the prism angle. Finally the change of index with *wave length*, $dn/d\lambda = 0.6 d\triangle/d\lambda$, about half the change in the deviation for the same change in wave length.

The chief first order errors in spectrometry are the division errors of the divided *circle*, amounting to from 2″ to 10″ of arc. These may be determined with mirror, telescope and scale, or with four-reading microscopes to about 0.1″, and if corrected for, the index may be determined to the seventh place.

Collimation gives another first order error in the ordinary method but if a small auxiliary spectroscope is attached to the slit, the same monochromatic light may be used in determining the two limits of the deviation and no change of focus is necessary in measuring each deviation. The error due to an error in wave length is of the first order, but if known spectrum lines are used as sources this error is negligible. If a section of a dispersed continuous spectrum (like that from a glow lamp) is used, an error of $0.1\mu\mu$ (1 Ångström unit) in the wave length corresponds to an error of about 0.00002 in n. The variation of n with temperature

is for glass of the order of 0.000002 per degree, with prism angle (as above) 0.7 of the variation in prism angle in radians.

Among the many second order errors may be mentioned those due to telescope and collimator not being coaxial, lack of parallelism of the beam traversing the prism, prism being slightly tilted out of the normal position, slight spherical curvatures of prism surfaces and most of the errors due to the thermal expansion of the instrument.

In the *photographic determination* of refractive index, an exposure is first made with collimator and camera arm approximately collinear, then the latter is swung through a known angle and some spectrum consisting of known lines is photographed. From the general deviation and dispersion the indices of the prism are computed for each wave length by the general formula, corrections being applied for inclination of the plate. Errors due to lateral chromatism and curvature of field in the camera lens may be quite serious in photographic refractometry.

In the autocollimation spectrometer of Abbe, the telescope itself serves as collimator, the light entering a slit just in front of the ocular and being reflected forward by a reflecting prism. A 30 degree prism is used, the light being reflected from its rear face. The instrument is simple and compact and very serviceable for five-place work.

Total Reflection Methods.

If a single plane face is the only optical surface available on a specimen whose refractive index is desired or if it is too opaque to transmit sufficient light through a prism, recourse must be had to one of the total reflection methods. These methods involve the use of an ordinary spectrometer and an auxiliary prism of high index whose indices are accurately known.

In the figure let X be the unknown medium and P a prism whose constants are known. Let light be incident at the common face parallel to that face. There will be refraction in the prism up to a certain limiting angle and again on

leaving the prism. Let n be the unknown index of the specimen X, n_o the index of the prism and n_a of the air. Then if $n_o > n$

$$n = n_0 \sin r_1$$
$$i_2 = r_1 - p$$
$$n_0 \sin i_2 = n_a \sin r_2$$

hence from r_2, n_a, n_0 and p, n may be determined.

From the limiting angle, in one direction, the field will be all light, in the other dark, which side is light depending

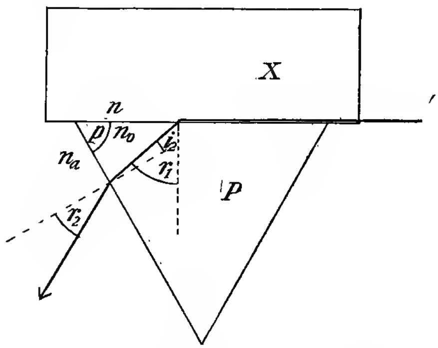

FIG. 30.—Refractive index of a solid having a single plane face

on whether the light is incident on the side of X or of P. The setting is to bring the edge of the illuminated portion up to a fixed reference cross hair. The method is sensitive (depending on n_0 approaching n and r_2 approaching 90°), but lacks precision from the difficulty in bringing a light field just into contact with a dark cross wire.

The total reflection method is applied to fluids by pressing a drop of the fluid between two prisms, the constants of one of which are accurately known.

Various forms of commercial refractometers having scales reading directly in refractive indices are on the market. These are total reflection instruments with a refracting prism of about 60 degrees and giving the index to one or two units in the fourth decimal place.

Interference Methods.

If two light paths are adjusted to equality by coincidence of interference fringes and the optical density of one path be varied, the path length must be varied to bring the fringes back to coincidence. Take the case of the ordinary Jamin or Michelson interferometer. Suppose a plate of glass with plane parallel sides, index n and thickness t, is introduced perpendicularly to one light path. Then to reestablish coincidence of fringes that path will have to be shortened $t\ (1-n_a/n) = s$, n_a being the index of the air or other ambient medium. This gives n in terms of a measured distance s and a measured thickness t.

Of these two quantities, s may be measured in light waves to about $\lambda/20$ or say 0.03 μ by simple counting and estimating the fraction of a wave or fringe. However t is measured directly with a micrometer and with an uncertainty of nearly 1μ. Hence for determining the index of say a plate of glass the method is hardly capable of giving the index to the fourth decimal place. The method is better applicable to gases where t may be made large and s small, and to differential methods where t need not be known with great precision.

Differential Methods.

In the determination of slight differences in refractive index, for instance in finding the salt content of water solutions, the CO_2 content of air, etc., differential methods of great sensibility are applicable. In the prism method, a series of hollow prisms is arranged with normal end faces so as to give nearly grazing incidence at each interface between known and unknown liquids. The deviation is measured first with all cells filled with the same fluid and then with alternate cells filled with the unknown fluid. From these variations the relative indices of the two fluids may be readily computed from the general formulas.

The interference method is particularly well adapted to the differential refractometry of gases. An interference refracto-

meter recently devised by Löwe[1] gives relative indices to one or two units in the eight decimal place. The refractive indices of the rare gases have recently been determined by the Cuthbertsons by a differential interference method requiring but extremely small quantities of gas.

Dispersion of Air.

Refractive indices are ordinarily determined in air and the results are relative to air. The refractive index proper is reduced to a vacuum by multiplying by the refractive index of air which is about 1.000293. The same may be said of all wave length determinations. The indices of optical glasses, minerals and liquids are ordinarily given uncorrected for air unless otherwise specified, and this is sufficient for practical purposes, since optical systems are computed to be used in air and minerals are identified in air. The index of air is, however, so great that other indices, specified to the sixth and seventh place must either be corrected to vacuum or else temperature, barometric pressure and humidity recorded.

The best recent determinations of the indices of air are those by Kayser and Runge[3] (1893), and by C. and M. Cuthbertson[2] (1910) given below referred to 0° and 760 mm pressure.

$\lambda \times 10^7$	$(n - 1)10^6$
236	321.6 (K & R)
255	315.5 (K & R)
285	309.1 (K & R)
325	303.3 (K & R)
420	296.4 (K & R)
443	295.2 (K & R)
486.1	295.11 (C & M C)
563	292.4 (K & R)
546.1	293.60 (C & M C)
579.0	292.98 (C & M C)
656.3	291.92 (C & M C)

These data refer to air dried but not freed from carbon dioxide. They may be referred to another temperature t and pressure P by the formula

$$(n - 1)_{t,p} = (n - 1)_{0,760} \times \frac{P}{760} \times \frac{273}{t}$$

that is by multiplying by the relative pressure and dividing by the relative (absolute) temperature. The dispersion given by Kayser and Runge's data is represented by the Cauchy formula,

$$n = 1.000\ 28817 + \frac{13.16}{10^{15}\lambda^2} + \frac{0.316}{10^{24}\lambda^4}$$

while the Cuthbertsons' data is represented by

$$n = 1.000\ 28854 + \frac{13.38}{10^{15}\lambda^2} + \frac{5.0}{10^{24}\lambda^4}$$

the wave length λ being in centimeters. The latter data is[4] better represented (to within the error in measurement) by the polar formula

$$n - 1 = \frac{4.6463 \times 10^{27}}{16125 - \nu^2}$$

where ν is wave frequency. This formula is then of the form, applicable to gases in general,

$$n - 1 = a\lambda^2/(\lambda^2 - b)$$

in which a and b are constants.

The equivalent index of air at 20 degrees, 760 mm. for white light used by Bessel in his formula for astronomical refraction is n = 1.000 291 608, this was deduced from the observed refraction.

The correction for humidity has been determined by Lorenz.[5] If v is the vapor pressure in millimeters of the water vapor present in the air then

$$n_v = n_o - 0.000041\ v/760$$

Representation of Data.

There are several graphical, numerical and algebraic methods of representing the data obtained in refractometry, each useful for certain purposes. Single indices for mere identification are usually made with sodium yellow light 5893 and recorded as such, although hydrogen red, helium yellow and mercury green have found much favor for this purpose on account of their intensity and convenience. The indices of optical glass have for twenty years been determined for the five lines A' (K, 7682), C(H,6563), D(Na, 5893), F(H, 4861) and G'(H, 4341) and these being convenient for calculation, interpolation to other wave lengths is seldom required. For data for photographic systems another line is desirable and Hg 4047 is much used. Thus four different sources are required, K, Na, H and Hg. The inconvenience of changing sources is obviated by using dispersed white light. In this way indices are readily determined to the fifth place at equal wave length intervals 400, 450, 500, 550, 600, 650, 700, 750 $\mu\mu$ convenient for computations.

There are three *graphical methods* of representing dispersion that are serviceable for indicating the character of the dispersion at a glance. The first and simplest method is to merely lay off the various indices of one substance on a linear scale. The indices of a second substance at the same wave lengths are laid off to the same scale adjacent to the first, a third beside that, and so on. A second method is the ordinary one of plotting indices as ordinates against wave lengths or reciprocal wave lengths as abscissas on ruled paper. Curves for substances having the same dispersion are parallel. A third method is similar to this, but gives higher precision. This is to plot as ordinates not indices directly but departures from some simple curve like $(\lambda - 400)(n - 1.50) =$ constant (Hartmann's form). By this method indices may be plotted and read off to a few units in the fifth place, while by the first two methods the uncertainty will lie in the fourth place at best.

A formulation of refractive index as a function of wave length is necessary for making precise interpolations and for determining and eliminating accidental errors in precision refractometry, say to the fifth, sixth and seventh places. For this purpose various dispersion and interpolation formulas are in use. The oldest and simplest of these is the Cauchy formula

$$n^2 = a + \frac{b}{\lambda^2} + \frac{c}{\lambda^4} + \quad - - -$$

This is a mere power series expansion formula and has not proven very satisfactory. The theory of dispersion indicates that the natural formula is of the *Sellmeier* type,

$$n^2 = a + \frac{b}{\lambda^2 - \lambda_1{}^2} + \frac{c}{\lambda^2 - \lambda_2{}^2} + - - -$$

in which $\lambda_1 \lambda_2 - - -$ are wave lengths at which strong absorption occurs. To this are sometimes added terms of the form $e\lambda^2 + f\lambda^4 - - -$ The *Ketteler–Neumann* formula

$$\frac{1}{n^2} = A\,\lambda^2 + a + \frac{b}{\lambda^2} + - - -$$

holds fairly well for optical glasses. The dispersion of air and other gases is well represented by the formula of the Sellmeier type

$$n^2 - 1 = \frac{a\,\lambda^2}{\lambda^2 - \lambda_o{}^2}$$

The *Hartmann* interpolation formula

$$(\lambda - \lambda_o)\ (n - n_o) = const.$$

although intended to represent deviations (wave length interpolations on spectrographs) rather than index, really fits dispersion data quite as well as any other three constant formula.

Any formula of course may be made to fit the data exactly at as many points as it contains constants a, b and c. However, the labor of computing these constants is almost pro-

hibitive when there are more than four or five. Probably the best method in practice is to start with a simple formula, make a solution for its constants and then make a graphical plot of departures of data from the formula, afterward adding further correction terms to the formula if desired. The application of least squares to dispersion curves has not been found serviceable.

Variation with Temperature, Pressure and Impurities.

Refractive indices vary by a few units in the fifth or sixth decimal place with a change of *temperature* of 1° C. Most temperature coefficients are negative, that is the indices decrease with rise of temperature, but some (calcite, *e.g.*) are positive and some (rock salt, quartz) even positive in the ultra violet and negative in the visible spectrum. The thermal variation of index appears to be a resultant of two opposed variations, an increase of index with rise of temperature due to increased atomic (or electronic) activity and a decrease due to the decrease in density caused by expansion.

Temperature must be reckoned with in all precision refractometry and in the design of some optical instruments. Indices to the sixth place are of no value unless determined at a fixed known temperature. In lenses, a given percentage change in $n-1$ causes the same percentage change (of opposite sign) in the focal length. Changes of temperature do not affect the shape of a lens or prism.

The temperature and pressure coefficients of gases correspond closely to the changes in density in the gas, the quantity $n-1$ remaining always proportional to the density. The temperature coefficients of fluids are as a rule large and negative. That for carbon bisulphide is about -0.0008 per degree C, for water about -0.0001, for bromo-naphthalin -0.0005. The temperature coefficients of the optical glasses vary greatly both in sign and magnitude, ranging from $+0.00002$ for heavy flints to -0.000002 for light crowns. Other temperature coefficients are roughly: fluorite $-$

0.000012, quartz −0.000007, sylvin (KCL) −0.000036, rock salt (NaCl) −0.000037. These numbers are merely illustrative as they vary widely with both wave length and temperature. Many of these substances were investigated by Reed[6] to as high temperatures as 360° C with special apparatus. Practically all the reliable data yet obtained may be found collected in the recent editions of Landolt and Börnstein's tables.

The *pressure coefficients* of index have been determined for a few fluids. In units of the fifth decimal place the increase of index for an increase of 1 atmosphere pressure is roughly for: water 1.5, CS_2 6.5, ethyl ether 6.8, ethyl alcohol 4.2, varying with both temperature and wave length. If the pressure coefficients of glasses are of the same order, a range of 5 percent in the barometric pressure would just affect the sixth decimal place.

The effect of *impurities* may be estimated by the formula for mixtures. If X parts by weight of a substance of index, n_1, and density d_1 be mixed with $100-X$ parts of another substance whose index is n_2 and density d_2, then n the index and d the density of the mixtures are given by

$$100\frac{n-1}{d}=X\frac{n_1-1}{d_1}+(100-X)\frac{n_2-1}{d_2}$$

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V.

THE EYE AND VISION.

Either the eye or a photographic plate is an essential part of every optical instrument, as essential a part and imposing as strict limitations on the performance of the instrument as the objective itself. Hence a knowledge of the optical properties of the eye and photographic plate is of vital importance in the design of optical systems. For example, the resolving power and aberration corrections of a system should be such as to give definition equal to the resolving power of the retina or plate receiving the images, while higher definition would not only be useless, but would frequently entail the sacrifice of other useful properties. Further, both eye and plate are, within limits, adaptable to the instrument and these limits should be known. In discussing the eye and vision we shall first outline the form and properties of the eye as an optical instrument and then describe the properties of the retina as a selective photometer, and outline the relations bettween these properties and the definitions of light and color.

The Properties of the Eye.

Light entering the eye passes in order: the *cornea*, a thin retaining wall, the lens shaped *aqueous humor*, the *iris*, a stop, the *crystalline lens*, a double convex lens of variable convexity and optical density, the jelly-like *vitreous humor*, and lastly impinges on the *retina*, a mass of microscopic rods and cones, end-on, partially covered by the fluid *visual*

purple. The radii of curvature, thicknesses, and indices of the media are tabulated below for a normal eye.

Radii of Curvature.		Thickness.	Refractive Index.
Cornea	+ 7.83 (front)	0.5	1.351
	− 7.33 (back)	(0.5–0.6)	
Aqueous Humor	+ 7.33 (front)..........	3.6 unac.	1.3365
	−10.0, −6.0	3.2 accom.	
Crystalline Lens	+10.0, +6.0	3.6 unac.	1.41 outside
	+ 6.0, +5.5	4.0 accom.	1.45 center
			1.437 equivalent
Vitreous Humor	+ 6.0, −5.5	15.9	1.3365
	−12		

The radii for the accommodated eye follow those for the unaccommodated. Dimensions are in millimeters. These vary with the individual by 10 per cent. or more. The structure of the normal eye is about as shown in the figure.

The cornea.—The refraction of entering light rays occurs chiefly at the front surface of the cornea, hence this is the chief factor in image formation. Its curvature is not affected by accommodation. Its thickness increases slightly (0.5 to 0.6 mm) from the center to the edge. The radius of curvature of the front surface varies in different parts of the surface by a measurable amount, but in the normal eye is sufficiently uniform over the effective central portion to produce good images. The refractive indices of the cornea and other eye media except the lens do not differ greatly from that of water (1.33) or dilute water solutions.

The aqueous humor gives optical contact between the lens and cornea and takes up most of the displacement of the front face of the lens during accommodation.

The iris is a stop serving chiefly to adjust the brightness of the images formed on the retina, this adjustment being entirely automatic. Although influenced by conditions of health, by drugs, and by the emotions, the diameter of the

pupil is not consciously (by act of will) adjusted to the brightness of the object viewed.

The size of the pupil depends not only upon the brightness of objects in the direct line of vision, but upon the total amount of light entering the eye obliquely as well. If we view a moderately illuminated surface through a cylindrical tube (say 5 by 50 cm) held close to one eye and carefully blackened a mat black internally, the spot of surface seen through the tube will after a time appear considerably brighter than the same surface seen with the unshielded eye.

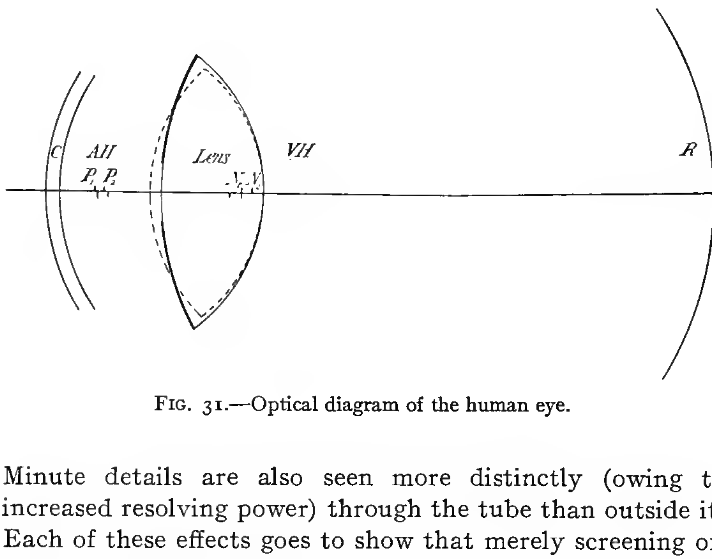

FIG. 31.—Optical diagram of the human eye.

Minute details are also seen more distinctly (owing to increased resolving power) through the tube than outside it. Each of these effects goes to show that merely screening off stray side light may increase the size of the pupil opening. No quantitative relation between the amount and direction of the light entering the eye and diameter of pupil has yet been determined. The diameter is roughly 2 mm with intense and 4 mm with moderate illuminations, increasing to about 7 mm for very faint illuminations. In most visual optical instruments a maximum of both illumination and resolving power are desirable at times, hence the importance

of effective eye shields in securing the maximum dilatation of pupil.

The position of the pupil is determined by the front surface of the crystalline lens against which it lies. The image of the pupil formed in front of the eye by the aqueous humor and cornea, is the entrance pupil of the optical system (see below) and since the position of the iris varies with the accommodation, so will this entrance pupil.

The crystalline lens is a thick biconvex body of firm but flexible material which under peripheral constriction (accommodation) increases greatly in convexity on its front surface and slightly on its rear surface. It shows cellular structure under the microscope, but is transparent throughout. Its refractive index varies from 1.41 in the outer layers to 1.45 at the center, the equivalent mean index being 1.437.

The limits of accommodation are by no means the 25 cm to ∞ ordinarily taken as the limits of the normal visual distance. The normal eye, even at middle age, can easily accommodate itself to an object beyond infinity, *i.e.*, a meter behind the eye, or as near as 10 cm in front of the eye. The power of accommodation decreases considerably, in practically all cases, from childhood to old age.

The vitreous humor is a jelly-like mass filling the space between the lens and retina, making optical contact between these and assisting in preserving the size and form of the eyeball.

The retina has a radius of curvature of about 12 mm The eye is unique among optical instruments in being provided with a spherical image surface, but makes little use of this since the eye is rotated to view distinctly objects at different angles.

From the data of the above table, the following optical constants of the eye (Helmholtz) may be calculated. The first column is computed for an object at infinity, the second for the eye accommodated to an object distant 25 cm.

	∞	25 cm
Front focus of cornea	23.27	23.27
Back focus of cornea	31.10	31.10
Focal length of crystalline lens	50.62	39.07
Back focal length of eye	20.71	18.69
Apex to first principal point	1.75	1.86
Apex to second principal point	2.11	2.26
Apex to first nodal point	6.97	6.57
Apex to second nodal point	7.32	6.97
Front focal length of eye	−13.74	−12.13
Apex to pupil image	−3.53	−3.14
Relative diameter of image and pupil	1.14	1.02

The two principal points are thus near together near the center of the vitreous humor, the two nodal points are also near together and situated just back of the iris in the crystalline lens. The back nodal point is the point of divergence of the back field pencils, hence determines the size of images and focal length.

Aberrations.—Of the seven third order aberrations, only the two axial aberrations, ordinary spherical aberration and axial chromatism, are of consequence in ordinary vision since only the axial portion of the retinal image receives our attention. These aberrations cannot be computed on account of the variable index of the crystalline lens, but experimental tests of resolving power show that both these aberrations are satisfactorily eliminated in normal eyes. No diffuseness nor colored fringes are perceptible at sharply defined edges of objects viewed that are as large as the half minute that is the limit of resolving power. No oblique chromatism is apparent (in the accommodated eye) even with objects of excessive contrast, but the effect would be masked to some extent by the decreased color sensibility of the lateral portions of the retina.

Resolving power.—No point or line, however fine, can appear to have an angular width less than $R = 1.2\lambda/D$

(roughly $0.0006/D$ where D = diameter of pupil in millimeters) on account of interference of the light waves. For $D = 4$ mm this limit is 0.00015 or about half a minute of arc. Since the back focal length of the eye is 20.7 mm and the index of the vitreous humor 1.3365, an angular separation R in an object viewed corresponds with an angular distance $R\ 20.7/1.3365 = R\ 15.5$ on the retina. For a 4 mm pupil, the resolving power corresponds with a distance of 2.2μ at the retina.

Illumination of image $I/I_0 = TSn^2/V^2$ for the eye, where T = total transmission (0.90 to 0.95 probably for most of the visible spectrum), S = area of pupil, $n = 1.3365$ and $v = 20.7$. In ordinary vision, S = about 10 sq. mm, hence the specific illumination of the retinal image is about 1/25 that of the object viewed.

The effective pupil of the eye, the external image of the actual pupil, lies in front of the cornea at distances respectively 3.1 and 3.5 mm for the accommodated and unaccommodated eye. The corresponding relative diameters of effective and actual pupils is 1.01 and 1.14.

The normal distance between axes of pupils is usually taken as 62 mm or 2 1/2 inches.

An eye focused on a bright light behind an observer is in just the condition to give a highly magnified image of the retina since the retina is just within the point conjugate to the observer. In this manner the retina may be examined effectively, the eye itself serving as magnifier.

The Retina.

The physical definition of light and color in terms of radiation, many of the fundamental principles of illuminating engineering, and many of the properties of optical instruments depend ultimately upon the properties of the human retina as a selective photometer. The relative sensibility of the retina to radiation of different *wave lengths* determines the relation of luminosity to radiant energy. Sensibility to

differences in *intensity* (contrast) is the basis of photometry, and the general integral of this sensibility gives the relation between the visual sensation of brightness and energy. Sensibility to *differences* in *wave length* of the exciting radiation gives the basis of all color scales and standards and of color analysis. The growth of the visual sensation with *time*, its lag and reaction (fatigue) determine persistence of vision and many related phenomena so important in flicker photometry, motion pictures, eye testing and other applications. Finally the *structure* of the retina is the ultimate limit of resolution in visual optical instruments and the differentiating functions of the rods, cones and visual purple are vital in many visual phenomena.

Structure of the Retina.—The retina is composed of microscopic rods and cones, closely packed together with their

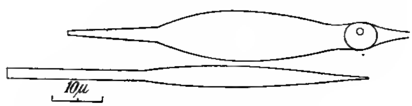

FIG. 32.—Rod and cone of human retina.

ends toward the center of the eyeball. Except in the central fovea, the rods are much more numerous than the cones and their inner ends form a fairly smooth, nearly continuous floor. The ends of the cones are considerably below the level of the ends of the rods (Fig. 32). The axial part of the retina, the *fovea*, receives the image given direct attention. The proportion of cones to rods is greatest in the fovea (10 : 1 or more) and decreases steadily outward to 1 : 10 in the outer regions of the retina (Fig. 33). The retinal size of grain (diameter of rods) is roughly 3μ (0.003 mm or 0.0001 inch).

Of the structure and functions of the optic nerve and the visual brain cells, which are as essential to visual perception as the retina, very little is known except that they are as adequate to resolve impressions as the retina and that through

them attention, memory, expectation and bodily fatigue may greatly affect visual perception.

The ends of the rods and cavities at the ends of the cones are filled with a watery blue fluid called the *visual purple.* Its function is not definitely established. It fades out and turns yellowish on exposure to bright light. It is abundant in the retina after rest in the dark, but scanty on continued exposure to light of moderate and bright intensity. During

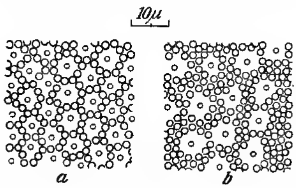

FIG. 33.—Inner surface of rods and cones showing distribution (a) central, (b) peripheral.

ordinary vision it is constantly being replaced or renewed from within the retina. Its amount and condition have much to do with retinal fatigue, sensibility and accommodation to lights of varying intensities.

Theory of Sensibility.—A physical instrument, responsive to stimulus, indicates in general the *quality*, *intensity*, *duration* or *extension* of that stimulus, and in some cases two or more of these properties at the same time. Simple instruments like chronometers and cathetometers indicate but a single property; most psychophysical receivers (sense organs) indicate all four simultaneously. An ideal physical instrument is entirely free from subjective variations (pressure, temperature, attention, fatigue, etc.) affecting its constants and producing systematic errors in its readings.

The three quantities *sensibility*, *scale reading*, and *stimulus* are simply related to one another. Sensibility is proportional to the derivative of the scale reading with respect to the stimulus and conversely the scale reading is proportional to the general integral of the sensibility expressed

as a function of the stimulus. We shall find these relations extremely useful in visual optics, when as a rule sensibilities (difference limens) are measured without difficulty, but scale readings (brightness, color, etc.) must be deduced from them.

Summary of Retinal Sensibilities.—1. *Spectral sensibility or Visibility.*—The retina is most sensitive to radiation in the blue green between wave lengths 0.50 to 0.55μ. Good seeing requires radiation between about 0.41 and 0.75μ. If sufficiently intense, radiation as far out as wave length 0.32 (ultra violet) or 1.0μ (infra-red) may be perceived. At low illuminations (rod vision) the maximum of visibility lies at about 0.51μ and does not vary with color blindness. At high illuminations (cone vision) the maximum of visibility is at about 0.54μ and varies greatly with color blindness.

2. *Photometric sensibility.*—Sensibility to differences in brightness is greatest over a wide range of moderate intensities and falls off toward low and toward high intensities. With decreasing intensities, it falls off more rapidly for red than for blue (*Purkinje phenomenon*).

The least perceptible difference in brightness (minimum contrast, difference limen), measured as a fraction of the whole is approximately (a) independent of intensity (*Fechner's Law*) extremes excepted and (b) independent of wave length (*König's Law*) extremes again excepted.

3. *Chromatic sensibility.*—The retina is sensitive to differences in wave length (which it perceives as differences in color), the sensibility varying irregularly throughout the spectrum. It is a minimum in the extreme red and violet, a maximum in the yellow and blue-green.

4. *Visual acuity*, so far as studied, appears to follow the same laws as photometric sensibility, namely approximately proportional to the brightness and independent of color.

5. *The growth and decay* of the visual responses with *time*, of which persistence of vision, sensibility to flicker, after images, etc., are special cases, are the resultant of a pure reception and a fatigue, each of which appears to be

related to brightness and wave length somewhat as is visual acuity and photometric sensibility.

Each of these retinal properties has been studied experimentally to some extent and sufficient quantitative data accumulated to determine roughly the forms of the various sensibility curves. Data has been determined on from one to twenty subjects in different cases. Normal eyes as a rule do not differ more than 10 percent from the means of a large number of eyes. In the quantitative summaries of data given below it must be borne in mind that reliable data is scanty in every case and that the curves given are but tentative at best. Only after data for a large number of eyes of individuals of several nationalities has been obtained can mean curves for the average normal retina be drawn.

In viewing any illuminated object the illumination of the image on the retina is about 1/25 that of the object viewed (see above). In retinal sensibility it is of course the illumination of the retina, not of the object that is of consequence, and since the pupillary opening is variable, it is customary in sensibility determinations to make use of an artificial pupil 1 mm^2 in area and to reduce all sensibilities to this standard. In all that follows relative to retinal illumination, the standard pupil of 1 mm^2 is assumed.

Spectral sensibility or Visibility.—The luminosity of a luminous body depends not only on the amount of the radiation received by the eye from the body, but upon its quality, the relative amount of each wave length or color. To deduce luminosity from radiation, not only must the spectral distribution of the radiation be known, but the subjective sensibility of the retina or *visibility* of radiation of each wave length as well. Luminosity is the product of spectral energy and visibility at each wave length. Visibility is the ratio of light to radiation, the factor which converts radiation into light or luminosity.

We are indebted chiefly to A. König[1] for data on visibility. In the accompanying table and figure are given the results of his determinations as recalculated:

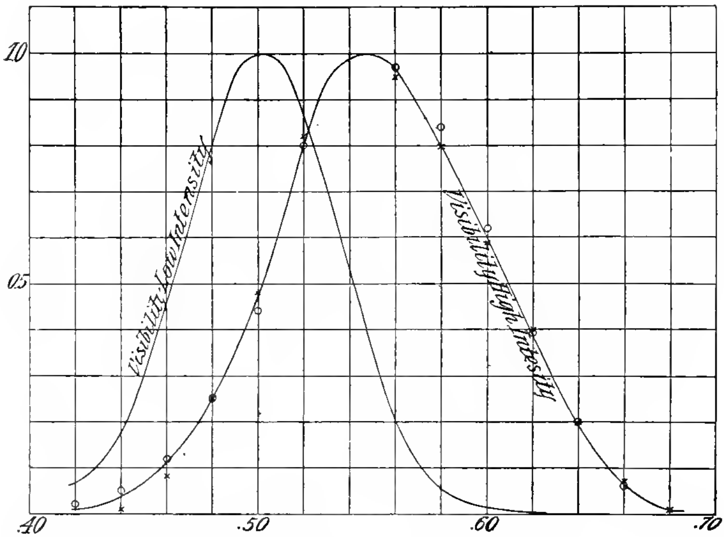

FIG. 34.—Relative sensibility of the eye to radiation of different wave lengths. o o, Ives' data.

Intensity steps....	T	A	B	C	D	E	F	G	H
Int. (m. c./mm^2 pupil).	.00024	.00225	.0360	.575	2.30	9.22	36.9	147.6	590.4
Ratio to Preceding Step.	......	9.38	16	16	4	4	4	4	4
430	.081	.093	.127	.128	.114	.114			
450	.33	.30	.29	.31	.23	.175	.16		
470	.63	.59	.54	.58	.51	.29	.26	.23	
490	.96	(.89)	(.76)	(.89)	(.83)	.50	.45	.38	.35
505	1.00	1.00	1.00	1.00	.99	(.76)	.66	.61	.54
520	.88	.86	.86	.94	.99	(.85)	.85	.85	.82
505	.61	.62	.63	.72	.91	(.98)	.98	.99	.98
555	.26	.30	.34	.41	.62	.84	.93	.97	.98
575	.074	.102	.122	.168	(.39)	(.63)	(.76)	(.82)	(.84)
590	.025	.034	.054	.091	.27	.49	.61	.68	.69
605	.008	.012	.024	.056	.173	.35	(.45)	.54	.55
625	.004	.004	.011	.027	.098	.20	.27	.35	.35
650	.000	.000	.003	.007	.025	.060	.085	.122	.133
670	.000	.000	.001	.002	.007	.017	.025	.030	.030
λ max	503	504	504	508	513	530	541	543	544

At low intensities (rod vision), the visibility curve has been determined for a number of subjects by König and Dieterici and by Pflüger using the threshold of vision method and by Langley using a visual acuity method, all of whom obtained results not differing essentially from those here given. At moderate and high intensities the visibility curves have been redetermined by Ives[2] using the best modern facilities and methods, again with confirmatory results. These data may then be used tentatively as a working basis. When other data based on means for

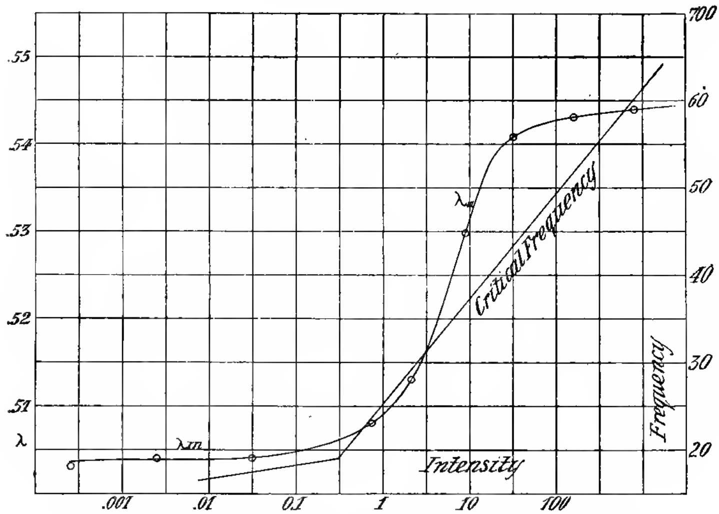

FIG. 35.—Wave length of maximum sensibility (λ_m) and critical frequency as functions of intensity.

hundreds of subjects shall be available it is unlikely that these mean curves will differ by as much as 10 percent or the positions of the maxima by as much as $5\mu\mu$ from those here given.

From the threshold of vision up to about one meter candle through 1 mm², there is little change in the visibility nor from about 40 m.c. to higher intensities. The

region of transition from rod to cone vision is from about 1 to 40 m.c. or from 0.1 to 4 m.c. for the ordinary pupillary opening of 10 sq. mm. In broad daylight and comfortable artificial illumination we are concerned chiefly with cone vision.

In Fig. 35 are plotted the wave lengths of maximum visibility against logarithmic intensity, clearly showing the transition from rod to cone vision.

Visibility may be formulated in the function

$$V = V_o e^{-\kappa(\lambda - \lambda m)^2}$$

holding well for rod vision with $\lambda_m = 504 \mu\mu$ and $\chi = 466$. For cone vision $\lambda_m = 544$, $u = 243$ and $V_o = 60$ candles per watt (about), but the formula does not hold nearly as well as for rod vision. The function proposed by Golhammer,[3]

$$V = V_o R^n e^{n(1-R)},\ R \equiv \lambda/\lambda_m$$

with $n = 126$ holds about as well for cone vision but not as well for rod vision (low intensities) as the preceding formula. The value of V_o for rod vision is unknown, but is probably much higher than for cone vision.

The visibility curve for cone vision is the resultant of three primary (blue, green, and red) visibility curves discussed later. Luminosity (spectral and total) is the integral of the product of visibility and spectral energy and is discussed in the chapter on Illumination.

Photometric Sensibility.—The least perceptible difference in light intensity decreases continuously with increasing intensity. Compared with other physical instruments the eye has a most extraordinary range. It may be used with ease at intensities a million times the minimum perceptible. It is comparable in action with an ammeter or galvanometer with an automatic shunt continuously variable.

The best method for determining photometric sensibility would be to deduce it from the mean errors of a large number of photometric settings at various luminosities. This would be very tedious and has never been attempted.

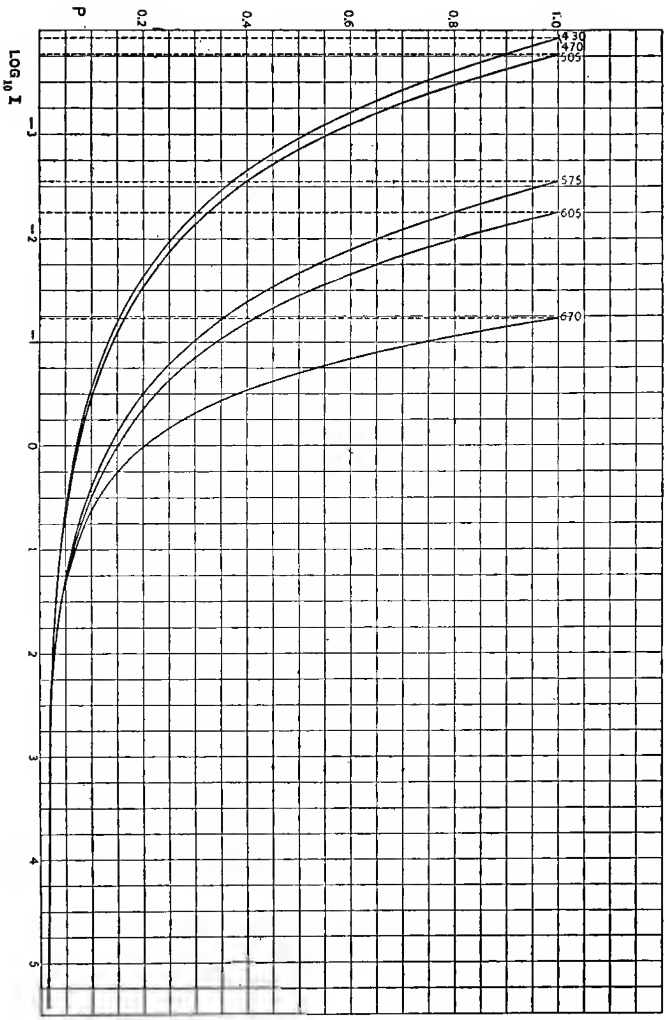

FIG. 36.—Photometric sensibility as functions of intensity and wave length.

The best data at hand is that of König and Brodhun[4] on the least perceptible increment (difference limen). They worked at six wave lengths 670, 605, 575, 505, 470, 430 and with white light, and over a range of intensities ranging from 1,000,000 down to 0.002 meter candles through 1 sq. mm. Starting with a moderately high intensity (600 m.c.) they worked to higher and lower intensities by multiples and fractions of these measured intensities. Their results are given in the accompanying table and figure. The limens are expressed as fractions of the whole intensity. The I_o at the top of each column is the threshold value measured down as a fraction of the high standard intensity.

$\lambda =$ $I_0 =$	.670 0.060	.605 0.0056	.575 0.0029	.505 0.00017	.470 0.00012	.430 0.00012	White 0.00072
I	δI:I						
1,000,000	....	....	....	....	....	....	0.0358
500,000	....	....	....	....	....	....	0.0273
200,000	....	0.0425	....	....	....	....	0.0267
100,000	....	0.0241	0.0325	....	....	....	0.0195
50,000	0.0210	0.0255	0.0260	....	....	....	0.0173
20,000	0.0160	0.0183	0.0205	0.0195	....	....	0.0175
10,000	0.0156	0.0163	0.0179	0.0181	....	....	0.0176
5,000	0.0176	0.0158	0.0166	0.0160	....	....	0.0179
2,000	0.0165	0.0180	0.0180	0.0175	0.0180	....	0.0181
1,000	0.0169	0.0198	0.0185	0.0184	0.0167	0.0178	0.0178
500	0.0202	0.0214	0.0180	0.0194	0.0184	0.0214	0.0192
200	0.0220	0.0225	0.0225	0.0220	0.0215	0.0245	0.0222
100	0.0292	0.0278	0.0269	0.0244	0.0225	0.0246	0.0298
50	0.0376	0.0378	0.0320	0.0252	0.0250	0.0272	0.0324
20	0.0445	0.0460	0.0385	0.0295	0.0320	0.0345	0.0396
10	0.0655	0.0610	0.0582	0.0362	0.0372	0.0396	0.0477
5	0.0918	0.103	0.0888	0.0488	0.0464	0.0494	0.0593
2	0.1710	0.167	0.136	0.0655	0.0715	0.0600	0.0939
1	0.258	0.212	0.170	0.0804	0.0881	0.0740	0.123
0.5	0.376	0.276	0.208	0.0910	0.096	0.0966	0.188
0.2	....	0.332	0.268	0.110	0.127	0.116	0.283
0.10	....	....	0.396	0.133	0.138	0.137	0.377
0.05	....	....	....	0.183	0.185	0.154	0.484
0.02	....	....	....	0.251	0.209	0.223	0.695
0.01	....	....	....	0.271	0.289	0.249	
0.005	....	....	....	0.325	0.300	0.312	
0.002	....	....	....	....	....	0.369	

The minimum value of the least perceptible increment P_m is about 0.016 (1.6 percent) for all colors. The curves are expressed fairly well by functions of the form

$$P = P_m + (1 - P_m)\,(I_o/I)^n$$

in which n has the values

$\lambda=$	430	470	505	575	605	670
$n=$	0.32	0.32	0.34	0.36	0.39	0.58

Hertzsprung calls the just perceptibly different intensities I_1 and I_2 and finds König's data for white light well represented by the function

$$-y = a + bx - cx^2$$

where $y = log\ log\ (I_1/I_2)$, $x = 1/2\ log\ I_1/I_2$, $a = 1.3138$, $b = 0.43595$ and $c = 0.05796$.

The Luminous Sensation, Fechner's Law.—Since, in general, sensibility is the derivative of scale reading with respect to the stimulus, we may obtain the visual sensation of brightness (scale reading), a quantity incapable of direct measurement, by integrating the sensibility function above.

In the above function P is expressed as a fraction $(P = \delta I/I)$ of the total intensity, so that sensibility, being inversely proportional to δI, is inversely proportional to IP. Let K be the constant of proportionality, then the luminous sensation of brightness B is K times the integral $dI$of $/IP$ or

$$B = \frac{K}{P_m} log\ (1 + P_m(I^n I_o^{-n} - 1))^{1/n}$$

the integration constant being zero since the sensation B approaches zero as the stimulus I approaches the threshold value I_0.

Fechner fifty years ago obtained the relation

$$B = K\ log(I/I_0)$$

which holds in the region of moderate intensities over which P is constant. It is a special limited case of the preceding function.

The Purkinje Effect.—If a red field and a blue field are illuminated to appear of about the same brightness and then the illumination of both greatly reduced in the same proportion, the red field will appear darker than the blue and conversely. This is because the ratio of the luminous sensation to the luminous stimulus (or its logarithm) is different for different wave lengths or colors. It is apparent from the data on both spectral and photometric sensibility. Through the violet and blue, 0.40 to 0.50μ, there is little or no wave length difference, but through the green, yellow and red, 0.50 to 0.70μ, the effect is large and increasing.

To obtain the amount of Purkinje effect we may take the ratios of the visibilities for intensities H and T in the spectral visibility table or the values of I_0 in the photometric sensibility table. It may be formulated either by taking the ratios of visibilities at high and at low intensities as given by the exponential formulas or by taking simply $P_m^{1/n}$ from the brightness formula. Either derivation gives $f(\lambda)\ log$ (Purkinje effect) = constant.

Physiologically the Purkinje effect is undoubtedly merely the effect of the transition from rod to cone vision. It is of vital importance in the photometry of light sources of different colors.

Chromatic Sensibility.—The visible spectrum is commonly spoken of as consisting of from three to seven distinct colors, blue, green, red, etc. Using dyed cards and carefully interpolating by means of rotating color disks, a series of about thirty color standards may be prepared, each just noticeably different in hue from its neighbors. Using spectral apparatus and the most refined methods the number of

distinctly pure different hues may be increased to fifty or more.

The fundamental quantity in color problems is the difference limen, the least difference in wave length perceptible as a difference in hue. The best data available in this field is due to Steindler[5] and relates to 12 subjects. The sensibility curves show two pronounced maxima in the blue green at 490 and in the yellow at 580$\mu\mu$ with two slight maxima

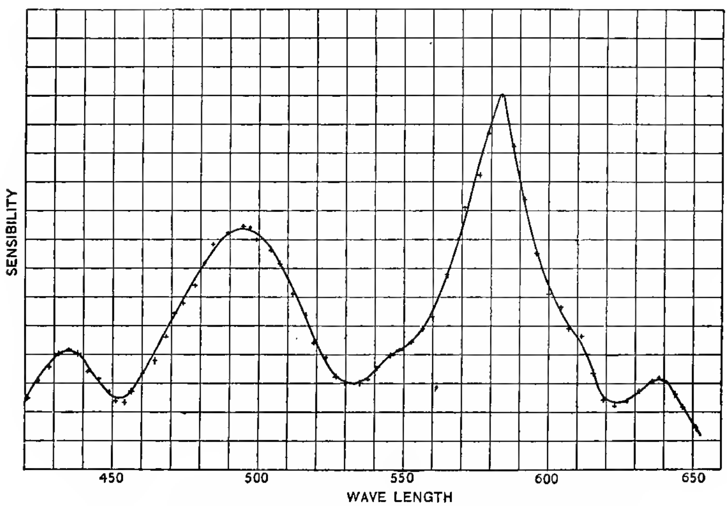

FIG. 37.—Sensibility to color differences. Steindler's eye.

in the violet at 420 and in the red at 620$\mu\mu$. The sensibility for various individuals varies so greatly that we cannot arrive at the desired properties of the average eye by taking a simple mean. These curves have been averaged by first determining the mean height and location of each maximum and minimum, and then connecting these seven points by a smooth curve. Steindler's data as thus reduced is given in the following table and figure.

	First Minima		First Maxima		Second Minima		Second Maxima		Third Minima		Third Maxima		Fourth Minima	
	λ	δλ	λ	δλ	λ	δλ	λ	δλ	λ	δλ	λ	δλ	λ	δλ
Dr. O. St....	435	23.6	454	37.6	495	11.6	535	32.8	585	7.6	626	40.0	638	31.0
Dr. E........	434	25.7	450	38.0	488	11.0	532	36.4	587	8.0	630	47.0	651	34.5
Prof. E......	430	14.5	444	18.0	480	5.0	523	34.2	586	9.0	624	34.2	637	28.6
Dr. Sch......	435	16.0	462	21.6	498	12.0	535	20.4	582	12.4	612	26.0	628	24.0
Dr. A. St.....	446	26.0	465	34.0	494	25.5	540	49.5	585	22.4	620	38.0	637	32.0
Dr. Ma........	....	....	....	....	488	15.9	520	22.0	572	9.1	622	24.0	638	17.2
Dr. Me.......	436	13.8	448	24.2	478	5.2	545	37.2	598	11.2	630	36.0	646	20.0
Dr. Bi........	447	18.0	462	24.0	505	12.0	540	22.2	583	11.2	608	21.2	614	19.2
Hr. B........	454	30.4	462	35.5	492	19.6	530	46.0	568	24.0	605	44.0	618	43.0
Frl. M........	442	14.0	468	24.4	490	16.0	540	32.0	572	20.0	633	60.0	642	45.5
Dr. H........	450	51.0	460	54.6	497	22.4	530	46.0	583	19.2				
Dr. G........	437	39.3	435	39.3	501	7.6	536	22.1	571	12.8	625	42.4	640	34.6
Means.....	440	24.7	455	29.3	492	13.6	534	33.4	581	13.9	621	37.5	635	30.0

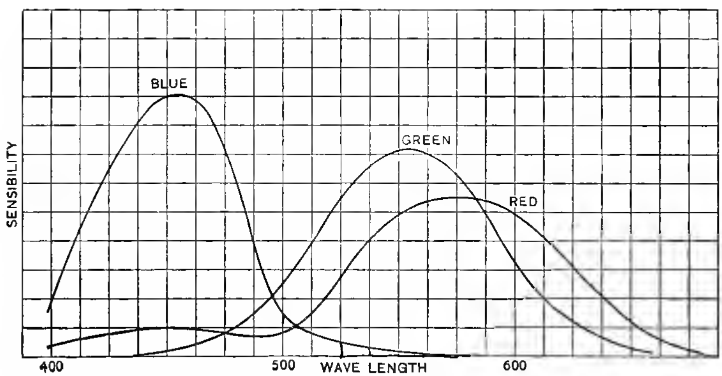

FIG. 38.—The three primary visual sensibilities.

Trichromatic Vision.—According to the commonly accepted theory of color vision, visibility at high intensities (cone vision) is a composite of three primary visibilities, blue, green, and red. These three primary sensation curves have been investigated, the curves given in the figure being

due to Exner.[6] Instead of being of equal area as here drawn their actual relative luminosities are according to Abney

Red	Green	Blue
65.7	33.8	0.44

The wave lengths of maximum color sensibility are at the points of maximum inclination to each other of the primary sensation curves.

Light which analyzes r percent red, g percent green, and b percent blue $(r+g+b=100)$ may be conveniently

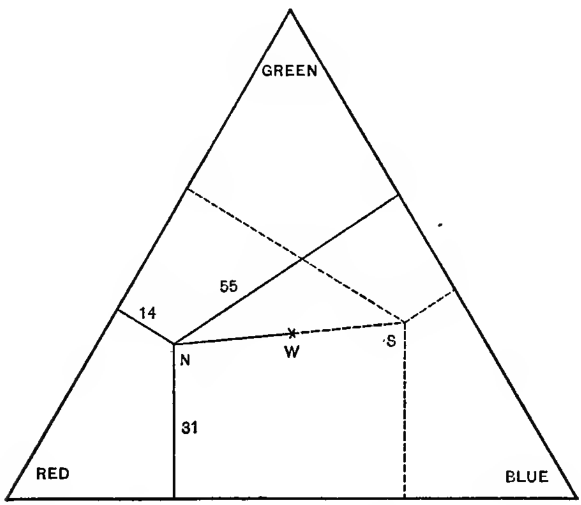

FIG. 39.—Color triangle.

represented by a Maxwell color triangle (Fig. 39) in which r, g and b are measured from the sides opposite the apices representing green, red, and blue respectively. Such light may be considered composed of white (center) plus a dominant hue. In the figure, for illustration are represented a light (N) of reddish yellow dominant hue and a second (S) which is its complement.

The Luminous Sensation and Time.

When light is thrown suddenly on the retina the visual impression lags somewhat behind the stimulus and is quickly followed by a fatigue effect lagging behind the impression. In the steady state the luminous sensation is the resultant of an impression and a fatigue. If the illumination of the retina is suddenly cut off there is a certain persistence of vision in the after image. There are various phenomena resulting from impression lag, fatigue and persistence of vision, some of which are of great practical importance.

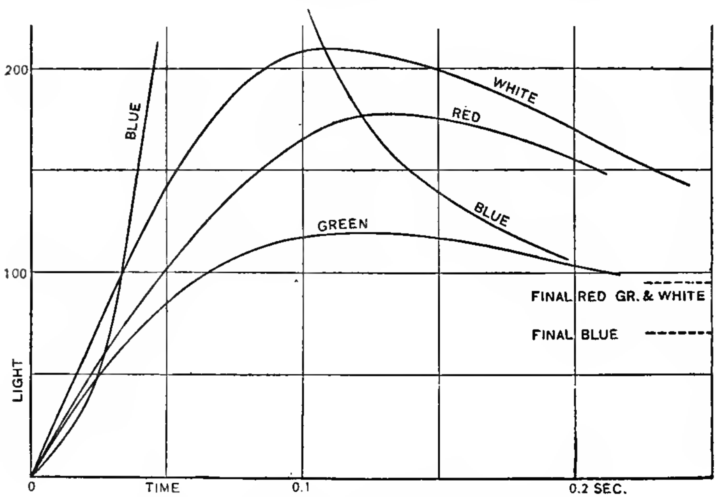

FIG. 40.—Growth of visual sensation with time.

With very low illuminations, the visual impression requires many seconds or even minutes to reach its full value; complete recovery of sensibility occurs in a similar interval. At ordinary intensities, however, the full sensation is attained in a few hundredths of a second.

Direct determinations of the growth of the visual sensation (impression plus fatigue) with time were made by Broca and Sulzer.[7] Their results were roughly as shown in the figure.

Working with red, green, blue and white light they found in every case an overshooting of sensation above its final value. With blue the maximum sensation was at least five times the final and occurred about +0.07 sec. after the initial exposure. Red and white overshoot to about double the final intensities after about 0.13 second. Green overshoots scarcely at all, indicating either very slight fatigue or else a very slight lag of fatigue behind impression.

The critical frequency of alternation of light and darkness at which flicker just disappears is related to the growth and lag of impression and fatigue. This has been investigated by Ferry[8] and by Porter.[9] Porter's data is summarized in the figure.

Critical frequency appears to be a function (logarithmic) of luminosity alone, being independent of color and practically the same for different observers. The plot consists of two straight lines, one evidently corresponding to rod and the other to cone vision and represented by

$$N_1 = 12.4 \log L + 29.4$$
$$N_2 = 1.56 \log L + 19.6$$

For ordinary intensities the critical frequencies range from 36 to 50 per second, hence they lie on the first slope of Broca and Sulzer's curves where they are nearly straight lines. At the critical frequency the mean values of the alternating sensations must be just less than the difference limen. Let $N = f(L)$ be the critical frequency for the luminosity L.

$$\frac{dB}{dt} = \frac{dB}{dL}\frac{dL}{dt} = \frac{K}{LP} LPnS = Ksf(L)$$

where S is the fractional sector opening and $f(L)$ is some function of L experimentally determined, for instance that found by Porter to be $a \log L + b$.

According to *Talbot's Law*, a periodic illumination (such as would pass through a rotating sector) will produce on the eye the same luminous sensation as the mean constant

illumination, provided the period is below that producing flicker. This law has been rigidly verified experimentally for white light by Hyde, but is yet without satisfactory theoretical foundation.[10]

Visual impression and fatigue may be represented by exponential functions of time, but experimental data is not sufficient to determine the constants of these functions. The rate of decay of the luminous sensation in after images has not yet been carefully investigated. The persistence of vision, as determined by critical frequency, has been utilized by Allen with great success in investigating color blindness.

Other Visual Properties.

All the visual sensibilities are subject to variations amounting to 10 or 20 percent or more with *attention*, *expectation*, *experience* (habit) and with *fatigue* and condition of *health*, both of the retina and the whole system. Visual *acuity* is roughly proportional to logarithmic intensity and independent of color. Both visual acuity and critical frequency have been proposed as rough direct means of photometry.

All the foregoing treatment has applied to the central part of the retina, the *fovea centralis*. At ten inches (250 mm) the area distinctly seen at one time is scarcely 2 mm. square, the remainder of the retina serving merely as a finder. The various sensibilities vary considerably from the fovea outward; *color sensibility* decreases considerably as does the *photometric sensibility* to intensity contrast. On the other hand, the *threshold limen* is considerably smaller in the outer parts of the retina. Extremely faint illuminations are more easily visible when viewed indirectly. Sensibility to relative continuous *motion* (of a small object relative to field) is nearly as great out from the center as at the fovea. Sensibility to slight *displacements*, difference in distance, bisections of short lengths, etc., is of course practically limited to the fovea. Sensibility to slight angles (lack of *parallelism*) amounts to

about five minutes of arc (1 in 700) at the fovea and is somewhat less in other parts of the retina.

Many optical *illusions* have been recorded and described but few if any of these are purely retinal in their nature. Their causes lie chiefly in various properties of the optic nerves and in the recording brain cells and their investigation belongs to psychology and psycho-physiology. Cases have even been recorded of (complete) color blindness due apparently entirely to a cerebral tumor.

Conditions for Best Seeing.

A number of the conditions for best visual acuity, comfort and efficiency have been mentioned and are here summarized. The chief of these are briefly: definition, size, illumination, contrast and stray radiation.

Definition.—The edges of details to be observed should be as sharp as possible down to an angular range of diffusion of about 1 in 10,000. It is useless to exceed this definition for the resolving power of the eye is only about 1 in 7000 or half a minute of arc for a diameter of pupil of 4 mm. This corresponds to (15.5/7000) but 0.002 mm or 2μ at the retina and the mean retinal grain is larger (3μ) and the retinal resolving power about 20μ. However, within this limit sharpness of definition is a great aid to acute, comfortable vision, particularly in viewing objects lacking contrast.

The best size of detail to be observed depends upon whether a casual survey, a rapid review (as in reading) or a minute inspection is to be given the object. Taking the middle ground, in reading we can read best with type subtending an angle of about 1:100. For comfort it should not be larger than 1:20 nor smaller than 1:300. Such data is useful in the adjustment of magnifications in visual instruments.

The *specific luminosity* of objects viewed, for best vision should not be over 0.3 nor fall below about 0.0003 candles per square cubic millimeter of surface viewed. In terms

of the illumination of white paper these limits are roughly 10 and 1000 meter candles and from 1 to 10 times these numbers for the illumination of colored objects. Inadequate illumination produces strain and fatigue where any degree of attention is required, too great is dazzling and overfatigues the retina. Diffusing screens are much used, even at great expense and waste of light, over artificial light sources merely to reduce the specific luminosity. The range of intensity accommodation of the eye itself is roughly 1:10 for the lids, 1:10 for the retina and with entire comfort at least 1:1000 in the retina. The quality of the illumination appears to be of lesser consequence except when a large percentage of highly fatiguing radiation of low luminosity (violet for example) is present.

Contrast in specific luminosity is best for vision at about 1:10 in intensity and without limit as regards color. Intensity contrasts should not be less than 1:100 for visual comfort, while contrasts as slight as 98:100 may be readily perceived with proper illumination. If objects are viewed through a screen, contrast will not be affected if either of the objects viewed or the screen itself is non-selective, but in special cases selective screens may be of great aid to vision. A light yellow screen is very effective in eliminating a purple haze, in viewing distant objects, and thus heightening contrast in an otherwise flat field.

Stray light and radiation of low visibility are important factors in visual acuity, comfort and economy. In viewing an object in a field more brightly illuminated than itself or with more brightly illuminated objects in the field, the pupil is contracted, lowering resolving power and retinal and nervous disturbances are set up which greatly interfere with distinct vision; even in the reverse case of brightly illuminated objects on a dark field. In the use of a shaded desk lamp for example, vision is not as comfortable as when the whole field of view is fairly uniformly illuminated.

Fatigue is a minimum in the green, yellow and red, but is greater in the blue and increases rapidly toward the violet

and still more rapidly in the ultra violet. At about 320 in the ultra violet the eye media become opaque and shorter waves may affect the cornea or even the pupil, but cannot reach the retina. The spectrum of daylight falls off rapidly from 420 in the blue violet toward the ultra violet and at 293 ends entirely. No injury to the retina nor lack of economy in vision is to be feared from the ultra violet of daylight nor of any artificial incandescent illuminant unless other parts of the spectrum are screened out. But the light from electrically conducting vapors like the arc flame, electric spark, vacuum tube and mercury arc, are as a rule, relatively much richer in violet and ultra violet radiation than the light from sources merely heated to incandescence and may be very injurious if unscreened or improperly screened.

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VI.

COLORIMETRY.

Objects are distinguished by their color as well as by their form and by their contrast in brightness with their surroundings. The light which objects emit, transmit, or reflect is characterized by its intensity, both absolute and relative, in different parts of the visible spectrum. The relative intensity of the various spectral components of the objective light determine the subjective interpretation, color, given it by the retina and brain.

To translate spectrophotometric results on spectral intensity into color by means of the trichromatic visibility curves and the color triangle is, at present, difficult and uncertain. In practical colorimetry, the general problem is the analysis and specification of colors and the precision must be such that any color may be recorded and reproduced to within the visual chromatic difference limen, that is so that no difference in either hue or shade is apparent to the eye. Such precision cannot at present be obtained by absolute methods and recourse must be had to simpler and more direct methods.

Nomenclature.—Colors differ in *brightness*, *purity* and *quality*. *Luminosity* refers to the brightness, *hue* to the position of the predominating quality in the spectrum. The spectrum is made up of a number of distinctly different hues. *Tone* refers to differences in colors all of the same hue, thus pink and claret are tones of red. Pink is spectral red mixed with white, claret with black. Mixtures of pure hues with white are commonly called *tints*, mixtures with black, *shades* of the hue in question. *Tone* includes both tint and shade. *Color* includes both hue and tone.

Specification and Analysis of Color.—Entirely aside from any theory of color vision, direct experiment shows that it is necessary to use three independent quantities in terms of which to specify color, and that three are sufficient to specify it without ambiguity. The three quantities may be the number of lumens of red, green, and blue which when mixed will match it (trichromatic analysis) or the wave length of the dominant hue, the amount of that hue and the amount of white, both in lumens, which compose the unknown color (monochromatic analysis). Color is frequently specified by giving simply the ratio red: green: blue in the first case or

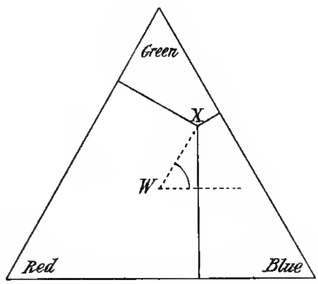

FIG. 41.—Monochromatic and trichromatic analysis.

wave length of dominant hue and percent white in the second; that is but two of the three quantities. This gives the nature of the color, but does not fix it. For example, straw yellow and a certain golden brown analyze the same proportion of red, green, and blue or the same wave length and percent white, but only the complete specifications in three variables fixes each.

These principles are well shown by the color diagram. Any color is representable by a point in an equilateral triangle. The *trilinear* coördinates of this point give the trichromatic specification of the color, the *polar* coördinates the monochromatic. The sum of the trilinear coördinates of every point in the triangle is the same. For corresponding points in triangles of different sizes, these coördinates are in

constant ratio. In practice it amounts to the same thing whether we specify the number of lumens in each component or specify the ratio red: green: blue and the side of the triangle or total luminosity. The straw yellow and golden brown giving similar analyses are represented by points similarly situated, but the yellow in a much larger triangle than the brown.

Color Scales.—The various hues, tones of those hues, and luminosities of both may be considered as arranged in graduated series with uniform intervals and definite fixed points.

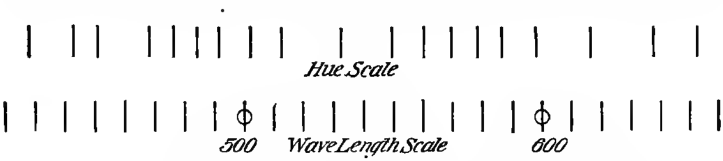

FIG. 42.—Hue differences corresponding to equal wave length intervals.

Such scales founded on correct principles and represented at convenient intervals with suitable reference standards are of great usefulness and interest.

Such scales are scales of subjective sensation, hence their intervals are proportional to the least perceptible differences in each case. No scales of hue or tone have yet been thoroughly worked out for lack of sufficient data on hue limen and tint limen. The scale of luminosity is simply the logarithmic scale corresponding to Fechner's Law. (See Chapter V).

The hue scale differs widely from a simple wave length scale, since visual sensibility to wave length differences differ widely in various parts of the spectrum. Coarse scales of wave length and hue are given side by side in Fig. 43 for illustration. The intervals on the (lower) wave length scale represent differences of $10\mu\mu$. The intervals are smallest in the hue scale in the region of the orange ($600\mu\mu$) and blue-green ($500\mu\mu$).

The direct method of setting up a hue scale is to start with a definite wave length, say orange at $600\mu\mu$ and lay off

a series of just perceptibly different hues by means of dyed cards. Ridgway chose first what he considered the purest red, orange, yellow, green, blue, and violet and then interpolated from two to six intermediate hues in each large interval, adjusting by means of a color wheel and plotted curve of proportions. He obtained in this manner 36 primary hues differing by about twice the hue limen. Six

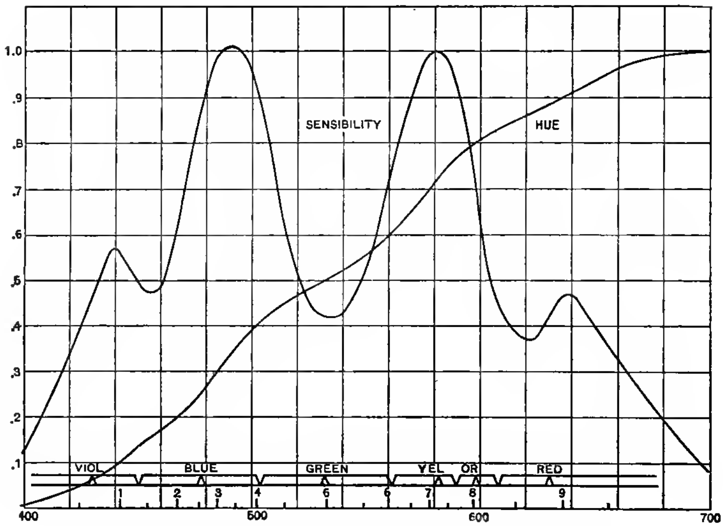

FIG. 43.—Sensibility to wave length differences. Hue scale.

of these are between violet and red. This method (v. infra) is based on sound principles, but to establish a general scale based on the mean sensibilities of a large number of subjects recourse must be had to more general methods.

The general method of establishing a hue scale is first to determine a curve of mean wave length sensibility at constant luminosity for a large number of individuals, then the integral of the sensibility curve is the hue scale desired. Steindler's data (Chapter V) on chromatic sensibility was not taken at constant luminosity so the resulting scale is

at best a good approximation. Values of the sensibility are taken at wave length intervals of $10\mu\mu$. These are added from the violet end and each partial sum divided by the sum total. This integration gives the ordinates of the color scale in decimal subdivisions.

Wave length	0.40	.42	.44	.46	.48	.50	.52	.54	.56	.58	.60	.62	.64	.66	.68	$.70\mu$
Color scale	0.01	.04	.11	.17	.27	.39	.47	.52	.60	.71	.81	.86	.91	.96	.99	.99

These values are plotted in the figure on a decimal scale. Any other subdivisions (20, 60, 100) might have been used

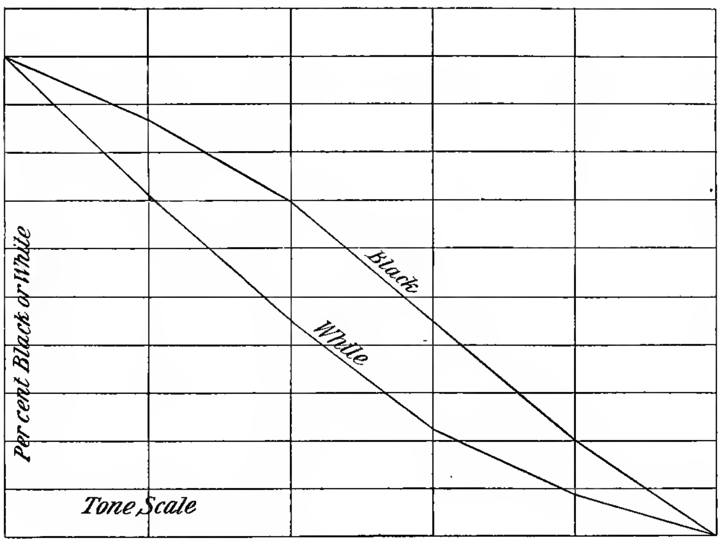

FIG. 44.—Rough tone scale.

equally well. There is no essential relation between the large intervals between the *named* fundamental colors and the small limen intervals.

There is but scant data on tone limens. The least perceptible increment of white or black is 5 to 20 percent

depending upon the purity of the color and slightly on the dominant hue. Laying off five steps from a pure hue to a pure white or black gives roughly:

White percent	0.0	9.5	22.5	45	71	100
Black percent	0.0	20.0	45.0	70	87	100

These values are shown plotted in the accompanying figure. When sufficient data on tone limen is available, the tone scale can be obtained like the hue scale by integration of the sensibility curve.

The number of different colors perceptible is of the order of 50 to 100 on the hue scale, 10 to 30 tints of each hue and an indefinite number of different luminosities depending on the upper limit chosen.

The terms, black, gray, and white are applied to mat surfaces according to their luminosity relative to their surroundings. A white surface dimly illuminated will match a gray surface brightly illuminated and either may, of course, match black.

What we call white (snow, white blotter, etc.) has a neutral reflecting power (albedo) of 0.7 to 0.8, gray about 0.4 and black 0.01 to 0.08.

Methods of Color Analysis.

All of the numerous so-called colorimeters are null instruments, known and unknown fields being brought to equality. Nearly all of them are merely color comparators the results being expressed in terms of more or less arbitrary reference standards, themselves requiring analysis.

Trichromatic Analysis.—In this method three primaries: red, green, and blue are chosen as elementary standards and then mixed in varying proportions until a match with the unknown sample is secured. The three primaries must be so chosen as to be capable of producing white or a neutral gray when mixed in the proper proportions. It is not necessary, however, that these proportions be equal (each 1/3)

nor is it necessary that the spectral curve of each or of any one of them be that of the corresponding spectral sensibility curve, (Fig. 45).

In the *Ives Colorimeter*[1] of the later type, light through three slits covered with blue, green and red glass is mixed by moving lenses, to match light directly from the unknown sample, the quantity of each primary being adjusted by varying the slit opening. The three scales each read 0 with the slit closed and 100 when open in the proportion to produce white. In the final match both color and brightness are made equal. An auxiliary white light slit permits a

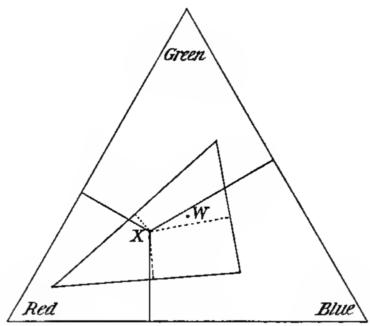

FIG. 45.—Secondary color triangle of colorimeter.

preliminary adjustment to equal brightness with a white unknown and each scale at 100. The instrument gives readings to from 2 to 5 percent under favorable conditions in the purer shades and more luminous hues, a precision barely sufficient for reproduction.

Similar results may be obtained with the trichromatic *rotating disks* of variable angle. Dyed blue, green and red disks, interlocking, are whirled on a spindle and the proportions exposed varied until a color match with the unknown is secured. The instrument is about as sensitive, as the Ives and far simpler, but the dyed cards are liable to fade while the Ives transmission screens are not.

Trichromatic *spectral apparatus* is simple in plan but very

expensive and difficult of manipulation and varations in the light sources used are very troublesome.

The readings of different trichromatic analyzers do not in general agree since the elementary primaries used are in general different. They may, however, be reduced to a common basis and to each other by means of a color triangle (blue 1/3 = green 1/3 = red 1/3). Each instrument has its

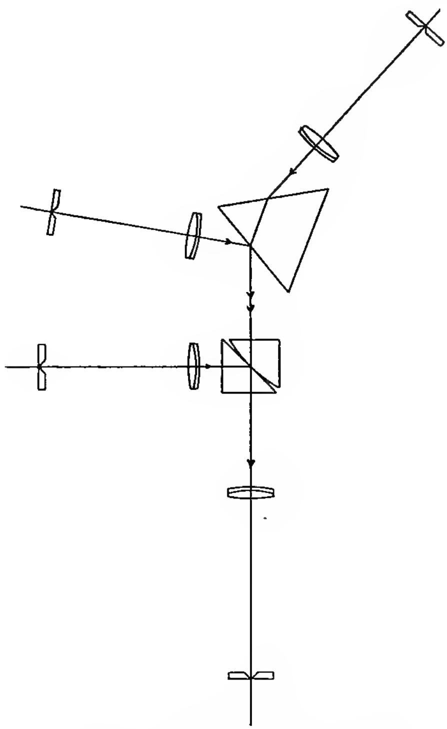

FIG. 46.—Monochromatic colorimeter.

own color triangle, as shown in the figure, which is not in general equilateral. But each instrument will determine the *same* point in the equilateral triangle if properly used.

Monochromatic Analyzers.—In monochromatic analyzers a

variable spectral hue is mixed with a variable amount of white and the luminosity of the whole varied to match the unknown sample. Here again as in trichromatic analyzers there are three variables and their adjustment is fully as difficult. In the color triangle, instead of three coördinates these instruments determine a length and a direction out from the white center. They may be of either the disk or the spectral type and until recently no servicable forms had been put on the market. The precision colorimeter designed and used at the Bureau of Standards is of this type, Fig. 46.

Interference Analyzers.—The varieties of color obtained by interference is sufficiently extended to match all unknowns and various instruments have been designed for practical colorimetry based on the interference principle. The best forms use a variable polarization to produce the color variation. In Brücke's "schistoscope" a cleavage plate[2] of mica is placed between a polarizing nicol and a Rochon prism. Meisling's colorimeter consists in a normal quartz plate between two nicols. As perfected by Arons,[3] a set of six quartz plates 1/4, 1/2, 1, 2, 4, 8 mm thick are used singly and in their 63 combinations, giving thicknesses from 1/4 to 15 3/4 mm in steps of 1/4 mm. Any color is specified in terms of a thickness of quartz and an angle between nicols. The method is simple and the instrument sensitive, but subject to systematic errors due to variations in the illumination of the white comparison surface and in the direction of the light through the quartz plates.

Analysis with Comparison Standards.—The utmost attainable sensibility and entire freedom from systematic error is obtained by directly comparing the unknown with prepared standards. The weakness of this method is the uncertainty of the color composition of the primary standards used.

In the colorimetric *chemical analysis* of compounds containing iron and a few other elements, the color of a solution of the unknown is compared with the color of a made up solution. Thus the color itself is not determined, but only used to estimate equality of composition. Remarkable pre-

cision in qualitative analysis is thus obtained. Many commercial instruments for the purpose are on the market. Various precautions must be observed in their use. The solutions should be in tubes having surfaces normal to the line of sight. The illumination should be ample, equal in the two tubes and free from either divergence or convergence in traversing the solutions. Finally the tubes with their contained solutions should be interchangeable.

Glass plates of various hues and shades are used as reference standards in the various *tintometers of Lovibond.* There are 20 to 60 shades in each set and different sets of many different fundamental colors adapted for dyes, sugar, leather, liquor, flour, oil, soap, water, varnish, gelatine, resin, etc. The instrument proper is merely a viewing box with racks for piling up the standard plates to secure a match. The readings obtained have of course little relation to any absolute scale nor can they be readily translated into those of any other instrument, but with proper precautions as to illumination, cleanliness of plates, etc., they give convenient reference points. With a single skilled observer in a single special line of work the instrument has proven serviceable.

In determining and specifying the colors of *mat surfaces* the set of reference standards developed by *Ridgway*[4] gives excellent service. These are a set of 1025 colored cards covering the whole gamut of hues and shades in just perceptible steps all carefully adjusted by the color wheel. Ridgway's six fundamental standards reflect narrow spectral regions whose dominant hues are; violet 412, blue 473, green 520, yellow 577, orange 598 and red 644, of which the violet, green and red are chosen as primaries. Between these six fundamental hues are interpolated 3, 4, or 5 other pure hues (making 36 in all including the fundamentals) to make the steps just preceptible from red to violet and through purple (5 steps) back to red.

The fundamental hues are then shaded off to pure white and black. The lighter tones are obtained by mixing (on

the color wheel) each of the 36 pure hues with 9.5, 22.5 and 45 percent of white. The darker shades contain 45, 70.5 and 87.5 percent of black. There are thus $7 \times 36 = 252$ primary hues and shades, aside from pure white and black. The secondary colors are derived from the primary by mixing each with neutral gray, this gray being the gray obtained by mixing the three primaries, blue, green, and red in such proportion (20 red, 32 green and 42 blue) as to produce a neutral tint. The proportions of gray with which the primaries are mixed are 32, 58 (36 hues), 77, 90 (18 hues) and 95.5 (6 hues) percent.

All these 1025 different colors, carefully adjusted on the color wheel, have been successfully reproduced in non-fugitive pigments and the collection is put out in book form. These standards, while arrived at by purely empirical methods, are yet on a thoroughly scientific basis in that each separate reference standard differs in either hue or shade from its neighbors by but little more than the least perceptible wave length or intensity difference.

In a series of color standards differing chiefly in shade but very little in hue, is sufficient for practical purposes in many cases. *Ashley's ceramic color standards* vary from buff to pure white and from bluish-white to pure white. They are obtained by mixing a white with a buff (or blue) clay in varying proportions (10:0, 9:1, 8:2 . . .) and represent nearly uniform shade steps. Such a series of standards may readily be calibrated with a simple reflection photometer in terms of coefficients of diffuse reflection. In using a photometer or a thin wedge of dark neutral glass to determine shades, color may be added to the white comparison standard or removed from the colored sample by interposing thin calibrated wedges of colored glass; in the first case of the same, in the second of the complementary hue.

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VII.

ILLUMINATION.

The principles of illumination are based partly on the properties of human vision and partly upon the production and distribution of light. Comfortable, acute vision requires abundant illumination of the proper *quality*, *distribution*, and *direction*. It is the special province of illuminating engineering to determine how such illumination may best be secured. The more physical principles of illumination relate to the amount, quality and distribution of light from *sources* and reflecting *surfaces* and of the light required for *vision*. These principles are here outlined, related subjects are treated under Photometry, The Eye and Vision, Colorimetry, and Spectroradiometry.

The quantities used in the study of illumination are those required to specify the amount, quality and distribution of the light given out by one surface or received by another (see Introduction). Those quantities referring to light differ from those referring to radiation in general only by a factor called *visibility* or sensation ratio, which is zero outside the visible spectrum and varies with the wave length within.

The fundamental entity here as elsewhere in optics is the *pencil* of light, through every cross-section of which the amount of light flowing is constant. If a screen be actually interposed in the pencil it will be illuminated by it, the *illumination* at any point of the screen being such as would be given by a point source of a certain luminous intensity, measured in *candles*, hefners, etc., at a certain distance from the screen, measured in meters or feet say. Illumination then is specified in terms of intensity and distance, usually in meter candles, foot candles, or meter hefners.

On the other hand, the emission of light from a surface is in all directions over a hemisphere. There is a certain *flux* of light out from the surface (measured in lumens, say). A small screen at a distance would be illuminated by the emitting surface as though each square cm were replaced by a certain number of candles. The light emitted over the hemisphere in *lumens* is π times the emission in candles per cm^2.

A surface both receiving and emitting light receives a certain illumination and shines with a certain *luminosity.* The ratio of total light reflected (in all directions) to incident light is the reflecting power or coefficient of reflection. The ratio of total transmitted to incident light is the *transmission*.

The Requirements of Vision.

Regarding the visible properties of the objects to be viewed as fixed (beyond control) we have first to determine what illumination is most suitable for viewing them and must consider intensity, quality, distribution and direction.

Intensity.—As regards intensity two distinct standards of illumination are recognized; *threshold illumination* (twilight, moonlight, street illumination) barely sufficient for perception, and *normal illumination*, sufficient for maximum visual acuity and discrimination. The luminosity of objects just visible is about .00002 m.c. if white, .002 if red, .0001 if green, and .000003 m.c. if blue or violet. These luminosities are the illuminations necessary to produce them times the reflecting powers of the objects viewed, these ranging from 80 percent to 5 percent or less.

Threshold illumination is such as to give luminosities ranging from the above threshold values up to about 0.1 m.c., an intensity just sufficient for reading ordinary 12 point print and about the intensity of the light given by the full moon.

Normal illumination is certainly less than that of full sunlight incident normally on white paper (100,000 m.c.)

and greater than one meter candle. For the best daylight desk illumination we choose about 1000 m.c. For daylight shop illumination, where the objects viewed are of much lower reflecting power than white paper as a rule, 3000 m.c. is chosen. For night illumination we choose about 50 m.c. for desk and perhaps 200 m.c. for shop illumination, each corresponding to roughly 40 m.c. The chief reason for choosing artificial illumination lower than daylight illumination lies apparently in the less uniform distrubution and consequent denser shadows and deeper low lights usually characteristic of it. Where artifical illumination is well distributed and there are no excessive contrasts to suppress we instinctly demand a higher illumination approaching daylight. In visual optical instruments with the central field illuminated and the remainder dark we choose the lower illumination for best acuity.

Sensibility to *contrast* (photometric sensibility) is greatest at about 1000 m.c. for white light as well as for all colors. It falls off to about half its maximum value (see Chapter V, König's data) at about 100,000 m.c. and at 10 m.c., and of course falls to a minimum at the threshold of vision. Since visual acuity depends largely, if not chiefly, upon contrasts in luminosity, our natural choice of luminosity for maximum acuity is the luminosity giving maximum sensibility. One thousand or two thousand m.c. is further probably as near an average value of luminosity in outdoor daylight as could be estimated, hence it is not surprising that the eye should show maximum contrast sensibility at this intensity. Luminosity contrast is of course independent of the total luminosity, since it depends only on the object illuminated.

It has recently been suggested that print in light letters on a dark ground would be easier on the eyes. Less visual purple would undoubtedly be used, but the retina would be operating at lower efficiency and in a well lighted room with light surroundings, conditions would be bad for comfortable vision.

Unsteadiness of illumination may be a source of great

discomfort. The effect produced varies with the frequency of the flicker, being a maximum at about 6 to 10 per second, but decreasing to zero at higher frequencies. The frequency of maximum flicker varies with the intensity. (See Chapter V.)

Quality of Light.—The best quality of illumination for maximum visual comfort and acuity is beyond question white light, while for correct color discrimination it is essential. White is difficult to define and its definition is more or less arbitrary. Subjectively, white is that color in comparison with which any other color will appear tinted. Objectively, the light of the midday sun (blue sky light excluded) reflected from a non-selective surface is white. White can be defined in physical units only through its spectral energy curve. In the accompanying table and figure are given the spectral energy values of those standards thus defined together with the visible spectrum of acetylene.

Wave Length	Solar Emission	Atm. Trans-mission	Sunlight	Per. Rad. 5000°	Emission Acetylene.	Nichols' Scale
0.40	170	0.540	0.780	0.666	5.8	0.190
.42	174	.580	.860	.737	7.7	.230
.44	176	.613	.919	.800	9.7	.270
.46	175	.645	.963	.855	12.5	.316
.48	171	.675	.986	.898	16.5	.363
0.50	167	0.703	1.000	0.932	21.7	0.420
.52	161	.728	1.000	.961	27.6	.484
.54	156	.743	.988	.984	34.8	.554
.56	149	.765	.972	.996	43.7	.629
.58	143	.780	.950	1.000	54.0	.704
0.60	137	0.793	0.926	0.993	66.3	0.783
.62	131	.805	.898	.986	80.5	.864
.64	125	.815	.867	.974	96.5	.926
.66	119	.825	.837	.962	112.8	.978
.68	113	.833	.804	.943	130.1	.992
0.70	108	0.840	0.773	0.925	147.0	0.986
.72	102	.847	.736	.803	163.8	.956
.74	96	.854	.699	.880	181.5	.924
.76	91	.858	.666	.861		
.78	86	.864	.634	.830		
.80	81	.868	.600	.800		

The data on solar emission and atmospheric transmission are from the work of Abbot and Fowle[1] at Washington as the mean of five years, 1903–7, the transmission being reduced to midday. The fourth column headed sunlight is the composition of midday sunlight at Washington reduced to maximum ordinate unity. In the following column is the standard white advocated by Ives,[2] the

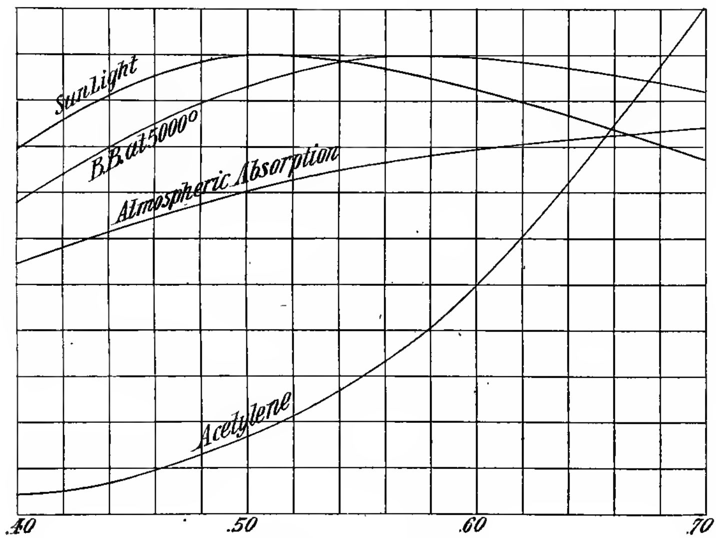


FIG. 47.—Energy spectra.

emission of a perfect radiator at 5000° C. absolute. In the final column is given data for Nichols' white[3] light scale deduced from his ratio of white to acetylene and the energy values of the latter obtained by Coblentz.

The radiator at 5000° would appear slightly yellowish in comparison with sunlight according to these curves. The product of radiation and visibility (Chapter V) gives luminosity. The luminosity curves for sunlight, perfect radiator at 5000° and acetylene are given in Fig. 49. The first two of these are nearly coincident.

The color accommodation of the eye is large, light departing very considerably from white with continued use may appear sensibly white. Acetylene, whose spectral energy distribution is given above, forms a very satisfactory illuminant as regards color yet its light is far from white.

The mere whiteness of illumination has little to do with its being agreeable. Ives has shown that light is pleasant to use when its color corresponds with that of a heated solid at some temperature whether this temperature be 1000 or 7000°. Illuminants like the Welsbach mantle, flaming arc, or mercury arc give light that appears harsh and disagreeable. In the color triangle their color lies off from the curve for an incandescent solid, but they do not differ from white as widely as some more agreeable illuminants.

The eye fatigue, strain and disorders caused by the continued exclusive use of some illuminants may usually be traced to an excess of the shorter wave lengths. Fatigue for a given intensity appears to be nearly constant through the green, yellow, and red, but increases toward the blue and rapidly in the violet and ultra-violet. The effect of ultra-violet appears to differ only in amount from that of the visible blue and violet.

Distribution.—For visual comfort we choose a nearly uniform distribution of illumination, a room with several windows or a skylight rather than one with but a single window. A uniform distribution gives minimum contrast (that caused by the varying reflecting powers) by minimizing shadows. Contrasts are usually ample (1:10) for distinct vision with abundant illumination and the most comfortable vision requires freedom from the heavy contrasts produced by dense shadows. In other words, the best illumination as regards distribution as well as intensity and color appears to be that of diffuse daylight out of doors, a decrease in contrast being produced by the elimination of shadows, but an increase of contrast due to high illumination and the correspondingly high photometric sensibility, maximum color contrast being secured by the use of white light.

Direction.—If limited in direction, illumination is most effective and comfortable if in the line of sight away from the observer, most disagreeable and ineffective if toward the observer. The first gives a minimum of harsh illumination contrast (due to shadows and specular reflection), the second a maximum.

Where the position of the observer and illuminator is subject to choice, the source of illumination (window, lamp) is placed behind or above the observer rather than directly in front.

Sources of Illumination.

As regards sources of illumination, we have to consider the amount, quality and distribution of the light supplied by them and the modifications in these produced by diffusing screens, walls and other objects, always with the end in view of satisfying the requirements of vision outlined above.

Intensity.—The amount of light emitted by a source (window, lamp, reflecting screen) is in general different in different directions, and different for different wave lengths of the visible spectrum. The total light emitted is the double integral of the light flux over a surface enclosing the source and for all visible wave lengths. It is expressed in quantity units in terms of some standard such as the candle, hefner, carcel, etc. It has no fixed relation to the energy radiated (watts), this relation depending upon the distribution of energy in its spectrum.

Consider first the intensity of the source as dependent upon the distribution of energy in its spectrum. Suppose spectral energy curves (E_λ) have been determined by radiometric methods for different parts of the source and in different directions from the source. Then the actual light emitted for each wave length is given by the product of energy (watts per unit wave length) and visibility (lumens per watt), $L_\lambda = E_\lambda V_\lambda$, and the total light is the integral of this product

over the entire visible spectrum. The visibility function V is of the form (see Chapter V)

$$V_\lambda = V_0 f(\lambda, \lambda_0)$$

where λ_0 is the wave length (about 0.54μ) of maximum visibility, $f(\lambda, \lambda_0)$ is a function which is unity at the wave length of maximum visibility and everywhere proportional to the visibility and V_0 is the ratio of light to energy (about 60 lumens per watt) at λ_0.

This multiplication and integration is in general performed graphically. In special cases E_λ is known and the product EV may be directly integrated. For the interior of a heated cavity with opaque walls (perfect radiator and absorber or black body) the emission at any temperature T (abs) may be well represented in the visible spectrum by the Wien-Paschen function.

$$E_\lambda = C_1 \lambda^{-n} e^{-c_2/\lambda T},$$

where $n=5$ and $C_2=14{,}500$. For heated carbon and metallic surfaces also this formula holds fairly well with n ranging from 5 to 7 in value.

Taking

$$V = V_0 M^\nu e^{\nu(1-M)}, \; M = \lambda_0/\lambda$$

the integral (L) of $EVd\lambda$ is of the form

$$L = A(B/T+1)^{-n-\nu+1}$$

in which A and B are constants. This is the light emitted corresponding to the measured energy E_λ. The total radiation emitted is the integral of $Ed\lambda$ which is $\alpha T^{n-1} = E$ say. Calling luminous efficiency $F = L{:}E$, the ratio of light to energy in candles per watt, F may be computed from the values of L, E and T above. For example we have for $T = 2000° = 1727°$ C, we have for

Cavity radiation	$n=5.0$	$F=0.055$ cand/watt
Carbon filament[4]	$n=6.0$	$F=0.209$ cand/watt
Osmium filament	$n=7.0$	$F=5.05$ cand/watt

showing the rapid variation of F with n.

By differentiating F with respect to T we find that for any body for which the above function E_λ holds, the luminous efficiency F has a maximum value

$$F_m = V_0 \left(\frac{n-1}{n-\nu-1} \right)^{1/2}$$

or about 10.8 cand/watt at a temperature

$$T_m = \frac{C_2}{(n-1)\lambda_0}$$

or about 6700° if we take $n=5$ and $\nu=126$. Thus a perfect radiator at best (6700°) would give only about 10.8/60 or 0.18 as much light as it would if all its radiation were confined to the region near 0.54μ in the green, and only about twice as much as an osmium filament at 2000°. The sun approximates to a perfect radiator probably about as closely as would a ball of carbon.

Aside from luminous efficiency, the remaining factor in the amount of light produced by a source is abundance of radiation. Intense radiation is secured by localizing the energy transformation. *Flames* set free large heats of combustion with carbon particles as radiators. *Glow lamps* provide a short path of high resistance and good radiating power. In the *arc*, *spark* and *vacuum tube*, the interposition of a short column of gas in the electric circuit provides an enormous local transformation of energy. Current and potential gradient provide innumerable electrons with enormous velocities and these give up their kinetic energy to radiating atoms or anodes which they strike.

In case the radiator is a heated solid (flame, glow lamp and ordinary arc), high luminosity is secured by high temperatures, thus decreasing the proportion of waste infra-red radiation. In the case of the metal filament lamps, an additional advantage is the high reflecting power in the infra-red as compared with the visible spectrum. The radiation from the Welsbach mantle and Nernst filiament is selectively weak in the infra-red. When the gases themselves are the

radiators (flame arcs, mercury lamps, vacuum tubes) high efficiency is secured by choosing a gas or vapor whose spectrum is as largely as possible confined to the visible spectrum; calcium, barium, mercury, titanium, nitrogen, carbon dioxide and neon are examples of those in use.

Quality of Illumination.—Maximum visual comfort requires light sensibly white and color discrimination requires a light that is not only sensibly white, but which has the spectral composition of white daylight. Next to white light the best illumination for visual comfort is secured by using sources whose color composition approaches that of a heated solid at some temperature between 2000 and 7000° (see Ives, above).

Colorimetric analyses of various illuminants have been made by Ives.[5] His results are given in the following table and color triangle. They are referred to the black body at 5000° as standard.

Source.	Color Values.		
	Red.	Green.	Blue.
1. B. B. at 5000°	33.3	33.3	33.3
2. Blue sky	26.8	27.2	46.0
3. Cloudy sky	34.6	33.9	31.5
4. Hefner flame	55.0	38.8	6.2
5. Carbon lamp 3.1 w.p.c	51.3	40.4	8.3
6. Acetylene	49.1	40.5	10.5
7. Tungsten, 1.25 w.p.c	47.9	41.1	11.0
8. Nernst lamp	49.2	40.7	11.1
9. Welsbach, 1/4 percent Ce	42.5	40.8	16.7
10. Welsbach, 3/4 percent Ce	45.2	42.0	12.8
11. Welsbach, 1 1/4 percent Ce	47.2	41.8	11.0
12. D. C. carbon Arc	29.0	30.3	40.7
13. Yellow flame Arc	52.0	37.5	10.5
14. Moore tube, CO_2	31.3	31.0	37.7

In terms of direct sunlight standard white, the 5000° standard is 36.3 Red, 33.3 Green, and 30.3 Blue, hence the

above values may be reduced to sunlight white by subtracting three percent from the red and adding 3 percent to the blue color values.

Nearly all the artificial illuminants are deficient in blue, having a dominant hue in the yellow. Several, however, are nearer the standard white than some ordinary forms of

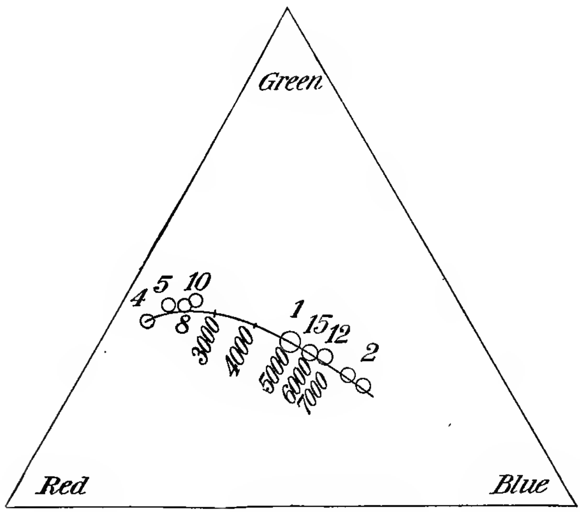


FIG. 48.—Colors of some illuminants.

daylight, namely the carbon arc, the mercury arc, and the carbon dioxide vacuum tube. The last lies extremely near to white light and its light is so evenly distributed through the spectrum that probably few color discrimination tests could be devised which would make its slight defects apparent to the eye. The dotted curved line represents the successive colors exhibited by a heated body at the various temperatures indicated. In the figure the white standard used is the sunlight white.

It is frequently desirable to obtain a white light from a colored illuminant. This may be accomplished in general in

three different ways: (1) by screening off the excess over white light, (2) by screening out the dominant hue and (3) by balancing up with an illuminant of the complementary dominant hue. The result may be either sensation white or spectral white depending upon the source and method used.

Sensation white may have an infinite variety of compositions so long as the resultant impression is that of white. The light from any source, whether its spectrum is continuous or composed of lines, may be reduced to sensation white by any of these three methods provided it is not limited to the extreme violet or red ends of the spectrum.

Spectral white is identical in spectral composition with standard white. Color discrimination requires illumination that is a very close approximation to spectral white. Only light having a continuous spectrum can be reduced to spectral white by either the first or third methods given. Of these only the first is practicable. Using say a screen to reduce lamplight to white light, if the screen has a thickness x and a transmission coefficient T at each wave length, the screen will reduce the lamplight to standard white if at each wave length the condition

$$\text{constant} \times \text{lamplight}/\text{white} = 1/(1-R)^2 T^x$$

is satisfied. R is the reflecting power of the surface of the screen.

Distribution.—The distribution of light (or radiation) about a source depends on the amount emitted by each element of its surface in each direction. The general problem of finding the distribution on any surface, of light from any surface with any distribution over the surface has never been solved. Some general properties of the luminous field, chiefly relations between equiluminous surfaces, have been developed by Hyde. Numerous special problems have been solved.

Consider a narrow pencil of light of solid angle ω along

which flows radiation W (watts) or light F (lumens), whichever we choose to consider. Then

$$\frac{W}{\omega} = \int_0^\infty E_\lambda d\lambda \qquad \frac{F}{\omega} = \int_0^\infty V_\lambda E_\lambda d\lambda$$

are the fundamental relations applicable.

Again consider any surface either emitting or absorbing radiation and integrate the flux over any other surface completely enclosing it. Then by Gauss' equation

$$F = \int_S f ds = 4\pi I$$

If the elementary flux f is taken positive outward, then I is positive or negative according as more radiation flows from or toward the active surface. I is the intensity of the source or sink of light (or radiation). If the flux is all outward, I is the intensity of the source in candles, watts or similar units. If the resultant flux is inward, I is the absorption; the ratio of the total outward to the inward flux is the reflection.

The practical units in use in illuminating engineering[7] are essentially as tabulated below:

Quantity	Symbol	Unit	Relations
Intensity........	I	Candle............	$I = F/\omega$
Flux............	F	Lumen	$F = I\omega = ES = \pi Q$
Illumination ...	E	Lumens/cm²	$E = F_i/S = I/r^2$
Radiation	E	Lux = Meter candle	$\acute{E} = F_c/S = \pi b = mE$
Brightness	b	Candles/cm²	$b = I/S \cos e$
Quantity	Q	Candles	$Q = bS$
Lighting	L	Lumen-hours......	$L = Ft$

The radiation emitted from a surface follows Lambert's cosine law more or less closely, that is the radiation from a given small area of surface in a given direction is proportional to the cosine of the angle between that direction and the

normal to the surface. In other words, the radiation into a given pencil is independent of the inclination of the pencil to the surface. Lambert's law holds closely for mat non-metallic surfaces, but departs several percent from the actual radiation for polished metallic surfaces. Assuming Lambert's law, the radiation[8] from a circular disk of radius a is $E = \pi b a^2/(a^2+d^2)$ at an axial point at a distance d from the center of the disk. The radiation from an infinite plane is independent of the distance from the plane. The radiation from an infinite linear or cylindrical source varies inversely as the distance from the source. The radiation from any surface is equivalent to that from its projected area.

Diffusing Surfaces.

Before being utilized for vision, the light from a source may be profoundly modified in intensity, quality, or distribution by reflecting or transmitting surfaces. Reflectors and diffusing screens are frequently used near artificial sources, the first to throw part of the light in a given direction, the second to reduce the specific brightness of the source. Window shades are ordinarily but light valves for controlling the amount of light entering a room, but may, if translucent become powerful secondary sources actually increasing the illumination. Wall, ceiling, and floor finish exert a profound influence on the illumination of an interior.

Transmission Screens.—Diffusing screens placed near sources of light reduce their specific luminosity to within the limits of comfortable vision (.0003 to .03 cand/cm^2), but at a considerable expense in light absorbed. In the following table are given the transmissions of various ordinary materials.

Clear glass,	0.92 down
Ground or frosted glass,	0.53 to 0.57
Light opal,	0.47
Diffusion glass,	0.32
Milk glass,	0.055 to 0.074

Smoke glass, light,	0.20
Smoke glass, standard,	0.38
Smoke glass, solar,	0.000,002
Paper, thin,	0.17 to 0.40
Paper, medium,	0.12
Same, oiled,	0.1
Paper, India and heavy,	0.08 down

Ground or frosted glass in different grades is very uniform in transmitting power. Diffusion glass is apparently clear glass filled with bubbles. In lamp bulbs some of the light lost during a single transmission is returned to the interior. Transmission screens emit about as nearly in accordance with Lambert's Law as prime sources.

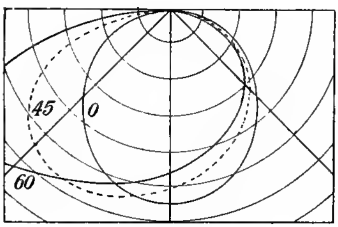
If a source of intensity I (in candles say) is enclosed by a spherical globe of radius R then the illumination of the interior surface, if uniform, is I/R^2 (lumens per cm^2). Then if the transmission of the material is T, the emission from the outer surface is TI/R^2 (lumens per cm^2) uniformly in all directions or $TI/\pi R^2$ (candles per cm^2) in a single direction. In practice the emission of the source is so far from uniform that such a calculation is of little value.

Selective transmission screens are used to modify the quality of the light for signals, for producing a white quality and other purposes. *Red* signal glass transmits about 80 percent of the red in the source and 5 percent of the total light. Green should transmit 50 percent of the total light. *Blue* signal glass transmitting all visible waves shorter than 0.50 cannot transmit more than 2 or 3 percent of the total light of ordinary sources. Bluish tinted glass is used considerably to reduce ordinary artificial light approximately to daylight.

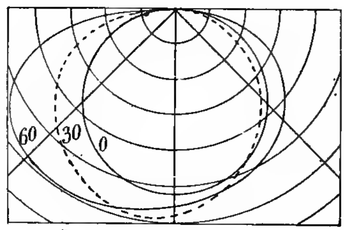
Reflecting Surfaces.—The reflecting surfaces chiefly to be considered are those of interior walls. The use of specular reflectors near sources is limited to the upper hemisphere in street illuminants for saving the light that would otherwise be wasted skyward. In interiors the use of such reflectors

would leave the upper half of the room but dimly illuminated. The better practice is to allow this light to go to the ceiling and reflect it back (70 to 80 percent of it) with a light paper or other coating.

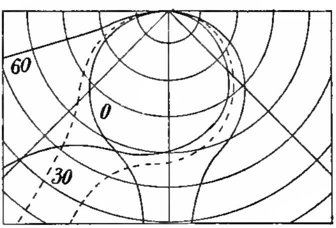
Lambert's cosine law holds very closely for mat reflecting surfaces. Whatever the angle of incidence, the distribution of the light reflected from a given area is in accordance with the cosine law. The reflected light is not polarized even in the extreme case of plane polarized incident light.



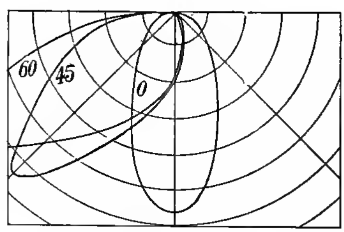
Smooth Matt



White Blotting Paper



Varnished Tile



Embossed Glass

FIGS. 49–52.—Reflecting power of papers at various angles.

Ordinary surfaces present all degrees of matness from purely specular reflection to perfectly mat surfaces as above defined. Neither these nor transmission screens have been extensively studied. Some recent work by Gilpin[9] well shows the character of the reflection from paper surfaces varying from white blotting paper to some heavily glazed. The figures show the distribution of the reflected light.

Fully half the reflection from the glazed paper appears to be specular.

In illuminating interiors, the sources of light must constantly supply light at the same rate that it is absorbed by the walls.[10] The amount absorbed in turn is proportional to the illumination, hence the light accumulates as it were until the light loss is equal to the supply. The supply in any given case is a direct function therefore of the reflecting power and area of the walls, but the former is so variable in most cases that only very rough results can be calculated.

In the special case of a spherical wall, the indirect illumination of the wall is everywhere equal in whatever part of the sphere the source be placed, or however unequally distributed its emission. A source of intensity I within a sphere of radius R whose walls reflect a fraction m of the light falling upon them produces an indirect illumination of the walls $(M\,I/(1-m)r^2$ If $m = 0.8$, then the total illumination (with source at center) is 5 times what it would be if the walls were perfectly black, $1/5$ is absorbed by the walls and the source just makes good the constant loss.

Non-selective (white or gray) reflecting surfaces vary in reflecting power from about 80 percent down to half a percent for black velvet. Selective reflectors have a similar wide range of reflecting power, of which part is usually due to actual absorption and part to spectral visibility.

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VIII.

PHOTOMETRY AND SPECTROPHOTOMETRY.

The amount of light emitted by a given source in a given direction is determined by photometry. Photometers usually determine the total flux in a pencil of light whose apex is at the photometer (screen) and whose base is the projection of the source in the direction of the apex. The exceptions are photometers for faint sources provided with lenses which throw real images of sources on the photometer screen. The flux in each pencil is determined by the illumination produced on a screen at its apex, this illumination being usually compared with that from a second source used as a reference standard and whose light value is known in terms of some reproducible primary standard. In this chapter are discussed a few of the more important photometers and photometric methods and the standards used in connection with them.

Photometers.

There are two chief classes of photometers, the comparative and the direct reading. Those of the first type are essentially null instruments, while the latter give values on an empirical scale calibrated in terms of some standard, but liable to subjective errors.

Comparative instruments are free from subjective systematic errors, but require the use of a reference standard at all times.

Direct Reading Photometers.—These are not as sensitive nor as accurate as the comparison photometers, but are simple and may be used under difficult conditions. One of the simplest of these is the *visual acuity* photometer. Some

simple ruled or printed test object is varied in angular size of detail until these details are just perceptible, under the unknown illumination. This method is, of course, applicable only at the lower illuminations, since at moderate and high illuminations it is either a mere test of resolving power (hence of pupillary diameter) or else (if of but moderate contrast) of photometric sensibility and neither of these varies much with illumination except when this is low. Another method of direct photometry is based on *sensibility to flicker*. The illumination is interrupted with variable frequency until the flicker just vanishes. This critical frequency we have seen (Chapter V) is proportional to the logarithm of the intensity. Both these psychological methods of course involve large subjective variations due to fatigue, experience, attention, etc., and must be used with care.

The *selenium* photometer and photoelectric cell as well as photographic photometry have found favor in certain classes of work, particularly in stellar photometry. With careful manipulation all these are sensitive and accurate, but their use requires skill and a thorough understanding of their properties. All have chromatic sensibilities differing from that of the eye, and the relative sensibility and the energy curve of the source must be known to reduce results to luminous values. The selenium photometer has been developed by Stebbins to a sensibility of about 1 percent at an illumination of the order of 0.003 m.c. With proper temperature control (an ice bath) for the receiving cell and other precautions the precision was raised to a corresponding figure. The *photoelectric cell* is sensitive, but has not been developed as a precision photometer.

Photographic photometry is extremely convenient in many cases and is particularly applicable to low intensities owing to its cumulative operation. It is subject to large errors due to developer, time of development, temperature of development, temperature of exposure, time between exposure and development, sensibility curve of plate, spectral

energy of source, and other variables, but these errors may be eliminated. The photometric sensibility of the plate is of course greatest at exposures in the steep central portion of the characteristic curve (see Chapter XI). Plates should be chosen for which this part of the curve is steep and if the photometry is to be at low illuminations, a plate of low inertia should be used.

In any case where a lens is used to throw a real image of a source on the photometric surface, the ratio of the illumination of that surface to the brightness of the source (in candles per cm^2 say) is TS/v^2 (see Chapter III). If the source is a point or nearly a point, the relative illumination of objective and an extra focal image is T, the transmission, times the relative area of objective and image.

The use of *selective screens* to modify the light to be measured in accordance with the chromatic sensibility of the eye has been attempted many times, but never with entire success. The difficulty lies partly in imitating the visibility curve in non-fugitive stains or solutions and partly the uncertainty and variability of the correction for reflection.

Comparison Photometers.—There are scores of different forms of photometers used for finding the relative intensities of two sources of light. All these are essentially null instruments, operating by reducing two illuminations to equality. Many of these are really successful instruments, working up to the full sensibility of the eye and simple in construction.

The illumination is equalized in a variety of ways; by varying the *distance* of one or both sources, interposing a rotating sectored *disk*, varying the *angle* of illumination of a plane mat surface, varying the intensity of the *reference standard*, varying the angle between two *polarizing prisms*, the width of an auxiliary *slit* or the thickness of an *absorbing wedge* or screen.

In comparison photometry attention must be given to (1) securing adequate *sensibility*, (2) eliminating systematic errors, (3) the use of a comparison source of fixed known value. The standard of attainment is in each case based on the pho-

tometric sensibility of the eye; the sensibility of instrument and method should considerably exceed that sensibility, systematic errors should be negligible in comparison with the least perceptible differences in intensity, and finally the values of the reference standard used should be known or obtainable to within the minimum possible error of photometric measurements.

Sensibility is secured by having the proper brightness and size of field for maximum sensibility and by bringing the two comparison fields exactly together without overlapping, the last being by far the most important factor. Color differences may produce large uncertainties, but do not greatly affect sensibility. From Königs' tables we see that the maximum sensibility occurs at an *illumination* of about 200 to 500 meter candles. At 5 m.c. the sensibility is about half as great as at the maximum. Illuminations even lower than this are often used in photometry, but at a sacrifice of sensibility.

The *size of the field* is of little consequence, but it may properly be limited to the size of the fovea, in angle about 4 1/2 degrees or 1 : 12. In one form of Lummer-Brodhun photometer one-half of one field is covered by a thin glass plate causing a slight (7 percent) loss of light by reflection and the opposite half of the other field by a similar plate. The setting is for equality of *contrast*. Neither theory nor its use have proven this form to have any considerable advantage in sensibility over the simpler form, but it is less fatiguing to operate. Binocular photometers are no more sensitive than monocular, but are preferred by some observers. Photometers with *ruled fields*, alternate bars being illuminated by the two sources, give high sensibility, but are very little more sensitive than those with simple bilateral fields.

The elimination of the *dividing line* between the two fields is the chief factor in photometric sensibility. Fields separated by but a narrow interval may differ by 10 per cent. and the difference be barely perceptible, while if further

separated so that the sensation must be memorized and carried from one field to the other, even for a fraction of a second, one field may be twice as bright as the other and the difference not easily perceived.

Of the early photometers the most successful in bringing the two fields together was the Bunsen grease spot photometer. The most complete elimination of the dividing line has been accomplished by means of total reflection prisms. The design of the Lummer-Brodhun cube is shown in the figure.

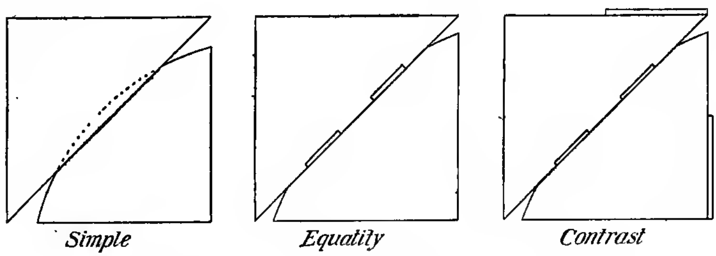

FIG. 53.—Forms of Lummer-Brodhun photometer cubes.

If the plane face joining the two prisms is well prepared, the shading off from one field to the other is but a few wave lengths wide and entirely imperceptible except under a high power microscope. This form of photometer head is now used more than any other in precision photometers and illuminometers.

Photometric sensibility is closely related to the *average deviation* from the mean in a series of photometric settings. Suppose photometric field A is just noticeably brighter than field B at a reading R_1, while at R_2, B is just perceptibly brighter than A. Then within the range from R_1 to R_2 any reading is as likely to be made as any other. If the setting is merely for equality such a series of readings would be represented by the flat topped curve (F) in Fig. 54. Skilled observers, however, as a rule first locate R_1 and R_2 roughly and then by touch make a final setting about half way between these two.

A series of readings by such an observer would be represented by the narrow curve (S) in the figure. Such a set of readings is apparently better than the first, but may be subject to a serious systematic error owing to a displaced criterion of equality or a false mechanical estimate of the median point.

Suppose the limen interval $R_2 - R_1$ has been determined by a large number of observations. Reduced to fractional intensities the mean value of this interval is the intensity difference limen determined by König (see V, Photometric Sensibility). Regarding R_1 fixed, the observed values of

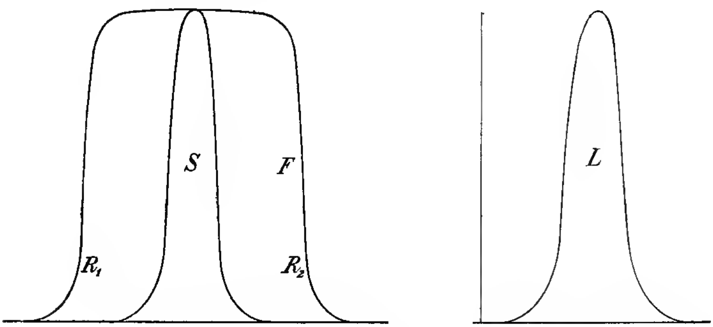

FIG. 54.—Photometric sensibility and probable error.

R_2 will lie on a probability curve as shown in curve L in the figure. The theoretical relation between curves L and F has never been worked out, but the mean width of F is not far from the mean abscissa of L. Mean deviations in a set of readings should therefore not exceed half the difference limen. Readings much farther from the mean than half the limen should be rejected as due to lack of attention (or worse).

In other words, with a difference limen of 1.5 percent, the average deviation of a number of settings should not exceed in intensity 0.4 percent. Readings departing from the mean by more than 0.8 percent may be rejected. The probable (accidental) error in the mean of 10 determinations should

not exceed 0.1 percent at most. Experienced observers have no diffculty in repeating readings to 0.5 percent with any good photometer, but the actual precision of such readings is doubtful until the method of setting and the personal equation of the observer have been investigated.

The elimination of *systematic errors* is much more important than sensibility in obtaining accurate results in photometry. The error due to a *false setting* was mentioned above. This error rarely exceeds 0.5 percent. Errors due to dissymmetry in the *photometer head* are usually eliminated by reversal of the head and taking the square root of the product of the two readings as correct. If the difference in the two readings is small this is equivalent to taking the arithmetical mean. Errors in measuring *distances* to the two sources may be large and in many cases troublesome. If reference standard and unknown are of similar character, such errors are eliminated by the method of *substitution.*

If the sources are rotating glow-lamps, the proper distance is from the surface of the photometric screen to the axis of rotation. In flames most of the light is usually emitted by the first two or three millimeters, so that even with thin flat flames the proper distance to be measured is subject to considerable uncertainty. If a screen is placed before a flame, the distance is of course properly measured to the plane of the screen. With unscreened cylindrical flames it is customary to measure to the axis of the flame, but this obviously gives too high a value to the source. *Glass surfaces* in the path of the light may, by reflection or refraction, very seriously affect the direction of the flux when not accurately plane. All of the other means of varying illumination listed above are subject to characteristic errors, with the probable exception of the rotating *sectored disk* of known angle. The validity of Talbot's law for such disks has been seriously questioned by some and firmly upheld by others. It seems probable that apparent departures from the law may be attributed to faulty testing. Errors due to a difference in color of the two sources may be serious, but hetero-

chromatic photometry requires special apparatus and methods.

Illuminometers.—Street photometers and illuminometers are essentially small portable photometers, enclosed and containing a small comparison standard source. They are designed to measure the total illumination on a given horizontal or other plane without reference to the number or positions of the illuminants. Scales are empirically calibrated by means of a known source. The chief requirement in these instruments is of course that the reference standard be sensibly constant for at least a few hours, and if possible constant or reproducible over long periods of time.

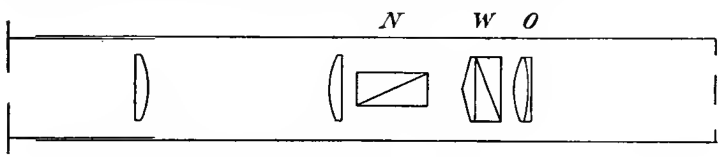

FIG. 55.—König-Martens photometer.

Certain forms of *polarization photometers* (really illuminometers) are extremely sensitive, free from sensible systematic errors and very convenient for determining coefficients of transmission and reflection for white light. One of the best and simplest of these is that devised by A. König and perfected by Martens. Light from the two fields to be compared is polarized and rendered divergent by a Wollaston prism *W*. This divergence is neutralized (and a little more) by the twin prism *P* which also serves to eliminate the dividing line between the two fields. The nicol *N* serves to balance the two fields, the relative intensities of which are proportional to the square of the tangent of the angle between the principal planes of the two polarizing prisms. The Ramsden ocular is focused on the apex of the twin prism. Auxiliary objectives giving actual images of the surfaces compared may be used if desired. The instrument as a whole is extremely sensitive and reliable.

Heterochromatic Photometry.—Where two sources differ in color it is difficult to compare their intensities by ordinary photometric methods. The eye is still sensitive to differences in luminosity in the two photometric fields, but is confused by the superposed color difference. An observer can make a set of readings in excellent agreement, but in wide disagreement with a subsequent set of readings or with a set made by another observer. These variations appear to be due merely to a shifting of the criterion of equality due to the confusing color difference rather than to any varying color sensibility, a cause frequently assigned to them.

The variable speed *flicker photometer* largely eliminates these uncertainties and gives true luminosity values. The two fields to be compared are viewed alternately in quick succession and a setting made for disappearance of flicker in the field, the frequency being adjusted[1] to the least for which flicker can be made to disappear. This method is applicable to such wide color differences as that between white and deep spectral red or blue, but with decreased sensibility. When the color differences are the comparatively slight ones of ordinary illuminants, the sensibility attainable is that of the best equality photometers; when color differences are wide, the sensibility falls to about 1/2 or 1/3 that value. According to Ives this minimum frequency varies from about 12 for yellow (compared with white) to 18 for green or red at moderate illuminations (25 mc.) down to about 8 or 10 per second for low illuminations (1 mc.). But little is yet known relating to the reproducibility of settings by the same observer or relative settings of different observers using the minimum frequency flicker method with large color differences.

Optical Pyrometry.—Several methods for the determination of high temperatures are photometric in principle, the temperature being deduced from the brightness of the heated surface. This brightness increases rapidly with temperature. According to the results of Nernst[2] the light emitted by a heated cavity at various temperatures is representable by $\log H = a - b/T$ in which $a = 5.37$ and $b = 11{,}230$.

This represents the maximum possible emission at a given temperature, the emission of exposed surfaces, particularly if bright, being considerably lower (v. infra).

Temperature.	Light.	Temperature.	Light.
1464° abs.	0.005 Hef./mm.[2]	2357° abs.	4.0 Hef./mm.[2]
1524° abs.	0.01 Hef./mm.[2]	2464° abs.	6.0 Hef./mm.[2]
1685° abs.	0.05 Hef./mm.[2]	2516° abs.	8.0 Hef./mm.[2]
1764° abs.	0.1 Hef./mm.[2]	2571° abs.	10.0 Hef./mm.[2]
1982° abs.	0.5 Hef./mm.[2]	2619° abs.	12.0 Hef./mm.[2]
2092° abs.	1.0 Hef./mm.[2]	2680° abs.	15.0 Hef./mm.[2]
2217° abs.	2.0 Hef./mm.[2]	2763° abs.	20.0 Hef./mm.[2]

The optical pyrometer of *Le Chatelier*[3] is a simple illuminometer or portable photometer for comparing the brightness of the heated surface illuminated by a small constant lamp. Absorbing red, green or blue glasses in the ocular confine the observations to limited spectral regions. Intensities are balanced either by varying the aperture of the objective with a sector or by interposing double wedges of neutral absorbing glass.

The *Wanner* pyrometer[4] is a modification of the König spectrophotometer. Comparisons are made at any desired region of the spectrum.

Intensities are balanced by rotating an analyzing nicol prism. Its weakness is, of course, the faintness of the spectra obtained at low temperatures.

In the *Holborn-Kurlbaum* and the *Morse* pyrometers the filiament of a small glow lamp is viewed against the heated surface. When matched, the filiament disappears against the background. The known current heating the filiament may be varied over a considerable range. For different ranges the light from the surface observed is varied by reflection or absorption.

In optical pyrometry the Wien-Paschen radiation function is used for interpolation and even extrapolation.

According to this the radiation E_λ in any short interval of the spectrum is the function $E_\lambda = {}_\lambda C_1{}^{-n} e^{-c_2/\lambda T}$ of wave length λ and temperature of radiator T. For sensibly monochromatic radiation this may be written

$$log\ E = a - b/T$$

where $a = log\ C_1 - n\ log\lambda,\ \ b = C_2/\lambda$

This formula is therefore sufficient for interpolation (and extrapolation) provided a and b (hence n and c) do not vary appreciably with temperature T. The above expression is identical with that used by Nernst (above) for the brightness of a heated cavity. It holds so well for total radiation because the radiation affecting the retina as light is nearly all (about 80 percent) confined to a narrow spectral region 0.1μ broad between 0.5 and 0.6μ. The formula applies to any higher temperatures or to different bodies for which n and C_2 remain independent of T.

In case a pyrometer has been calibrated on one body at known temperatures and it is desired to obtain the true temperatures of another body we have

$$log\ \frac{E_1}{E_2} = a_1 - a_2 - \left(\frac{b_1}{T_1} - \frac{b_2}{T_2}\right)$$

or in the special case of negligible variation in n and C_2, the formula of Rasch,[5]

$$log\ \frac{E_1}{E_2} = c\left(\frac{1}{T_2} - \frac{1}{T_1}\right)$$

This formula is used also to obtain the true internal temperature T_1 from pyrometric observations of the surface at a lower temperature T_2.

Photometric Standards.—In the determination of the actual intensities of light sources, one of the most important factors in obtaining precise results is the value of the reference standard used. The value of this *working standard* must have remained fixed since it was last compared with a set of *secondary standards* or until it is next compared with

them. These secondary standards must in turn remain constant over the longer intervals of time between comparisons with *primary standards*. The primary standards themselves must be reproducible from specifications.

The intensity unit is at present the *international candle* in England, France and the United States and the *Hefner* in Germany, the latter unit being 9/10 the former. These units are established and preserved by sets of ordinary glow lamps, both carbon and tungsten, filed away by the various national testing laboratories, intercompared and added to from time to time. Such lamps have been found to be extremely constant in value if properly seasoned and operated always at the same watt input. Their values are determined to about 0.1 percent and their variation is so slight that, barring accidents, they might serve for a century.

Many forms of primary standards have been proposed, but none of these are reproducible from specifications to 1 percent or better. Until some form shall be developed that shall be reproducible to within the uncertainties of intercomparisons (0.1 percent) the unit will continue to be defined and preserved in terms of secondary standards.

Up to about 1885 the standards in use were candles burning with a certain consumption and height of flame and prepared in a certain way. Then the *Violle Standard* was proposed and adopted, namely the light emitted by 1 sq. cm of platinum at its melting point. This has never proven satisfactory as regards reproducibility and is difficult to manipulate. The *Hefner* standard is a small lamp burning amyl acetate with a wick and open flame resembling a candle. With construction and flame height according to detailed specifications, this standard is reproducible to within about 1 percent, but its reddish hue is objectionable.

The *carcel* standard is a lamp with cylindrical wick and chimney, burning colza oil. It is difficult to control or reproduce and has found favor only in France.

The *pentane* or Harcourt standard burns a mixture of pentane vapor and air obtained by passing air over pentane.

The color of the flame is satisfactory and the construction, though complicated, can be duplicated by any skilled mechanician to give results within about 1 percent. It is the favorite standard in Great Britain. In the United States the pentane and Hefner standards are in about equal favor.

All flame standards vary with luminosity and barometric pressure, and to sensibly the same degree as ordinary gas flames. For this reason they are preferred for gas photometry, both the Hefner and pentane standards being in common use as comparison standards.

The only primary standard yet attempted that is entirely unaffected by atmospheric variations is the tube of conducting helium gas. This is as reproducible as any of the flame standards, but its color is reddish like the Hefner.

Aside from their reproducibility, the primary standards in use lack usefulness in that they differ widely in color from many illuminants. This necessitates stepping over a wide color difference either from working standard to secondary standard or from the latter to primary standard. It is possible that the flicker method may be developed sufficiently nearly to a pure luminosity basis to overcome this difficulty.

The simplest and most logical light standard is a radiation flux standard. The specifications for such a standard would be a certain amount of radiation in watts per sq. cm of either two or three specified wave lengths falling on the photometric screen and observed as light. Such a standard is in theory absolutely fixed and reproducible to within the uncertainty of measurement of the radiation as energy. This measurement cannot be made to much better than 5 percent by any methods at present available.

Spectrophotometry.

By the methods of spectrophotometry two sources of light are compared visually, wave length by wave length throughout the entire visible spectrum. By means of

photography similar methods and instruments are applied to the ultra-violet region of the spectrum. By the slightly modified instruments and methods similar comparisons of radiant energy are made in the visible and infra-red using as receiver not the eye, but some form of radiometer.

A quantitative comparison of two spectra point by point is essential in the study of selective emission, absorption and reflection, the relative intensities of the two spectra at each wave length being a measure of the relative emitting, absorbing or reflecting power of bodies. If the spectral energy (in watts/cm^2/unit wave length) be known for any source that of any other source may be determined in the visible spectrum from it by spectrophotometry.

The method of spectrophotometry is briefly to select a strip of the photometric field with a slit and disperse it by some form of spectroscope. In most common forms, the upper half of a slit of a spectroscope is illuminated by light from one source while the lower half is illuminated by light from the other source. In other forms (Brace and Lummer-Brodhun) the lights to be compared enter two different collimators and are afterward combined by a reflecting prism. Each form has its defects and advantages and for certain kinds of work certain kinds of instruments are preferred.

Diagrams of some of the best spectrophotometers are given in the accompanying figures. The Lummer-Brodhun (Fig. 56) gives a large clear sensitive field and is esteemed for comparing two different sources. Intensities are varied either by varying slit widths or by a rotating sector. It is subject to large and troublesome errors due to stray light. The König spectrophotometer improved by Martens is essentially the König-Martens polarization photometer with the addition of dispersing apparatus. It is sensitive and free from sensible systematic errors. It is well adapted to the determination of selective absorption and reflection since both beams may be derived from the same source and hence fluctuate in the same proportion.

For the varied requirements of an optical laboratory it is convenient to have a photometric attachment that may be used with any spectroscope with high or low dispersion as desired. The simplest device of this nature is a reflecting prism or rhomb (Hufner type) attached to the slit.[6] This though free from errors lacks sensibility. Thovert[8] and Ives[7] use a double reflecting prism to combine the light

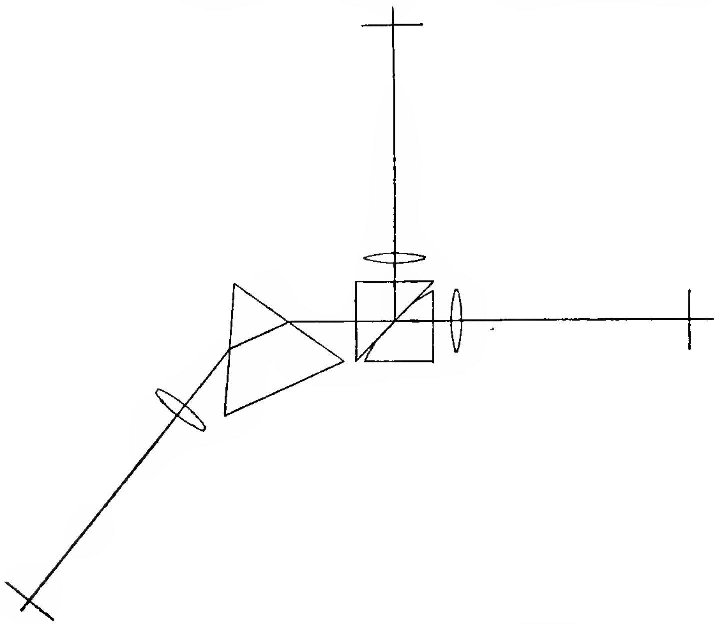

FIG. 56.—Diagram of Lummer-Brodhun spectrophotometer.

from the two sources. The second reflecting surface is silvered half way across. A weak lens placed at the slit of the spectroscope forms an image of the edge of the silvered surface in the neighborhood of the spectroscope prism, when it is viewed in monochromatic light. The polarization attachment utilizes a similar pair of reflecting prisms,[9] but the silvered surface is in fine strips 1/4 mm wide and an image of that surface is thrown on the slit of

the spectroscope by an achromatic lens. This instrument may be attached to any spectroscope, even an echelon.

Sensibility.—Sensibility in spectrophotometry depends upon precisely the same factors as in ordinary photometry namely (1) having the two fields to be compared close together with no dividing line of finite width, (2) sufficient but not too great intensity, and (3) a field of view of the proper angular size.

The fields are brought into close juxtaposition by totally reflecting prisms, biprisms, or silvered strips provided images of these are at the proper visual distance or within the range of accommodation of the observer's eye. The double contrast field is frequently employed in the Lummer-Brodhun instrument, but this and the finely ruled fields have a sensibility slightly, if at all greater, than the simple half field. The luminosity of the fields to be compared should be at least 30 m.c. if possible. It is often difficult to obtain sufficient luminosity for maximum sensibility in spectrophotometry, particularly at the blue end of the spectrum or if examining diffusely reflected light. The field viewed is best about 5 degrees in diameter. There is little if any advantage in a larger field, but with a field subtending but 1 or 2 degrees sensibility is lowered.

Sources of error in spectrophotometry are many and serious. First of all, the treatment of the two beams to be compared must be symmetrical in the instrument, otherwise troublesome empirical corrections must be applied. Lens and prism surfaces may easily cause unsymmetrical absorption by becoming dirty unless properly disposed. All stray light must be carefully eliminated or rendered symmetrical. Colored glass at the ocular will assist in its suppression.

If two sources are being compared it is essential that these either vary together or are both free from fluctuation. If the reflecting power of two mat surfaces are being compared these should be plane and inclined at precisely the same angle. Similarly in studying transmitted light, irregular refraction during transmission is to be guarded against.

In polarization instruments no variation in a variable azimuth should occur adjacent to any oblique reflecting surface. The most serious source of error is partial polarization of the light used. All movable nicol prisms used must of course have end faces normal to these sides. The Glan-Thompson form is the one commonly used. Good nicols require no intensity corrections at any azimuth.

If a line spectrum is compared with a continuous spectrum, the slit width correction applies to the latter, but not to the former if the dispersion used is prismatic. This correction, arising from the variation in the dispersion throughout the spectrum, may be computed from the optical constants of the prism or experimentally determined with ocular slit and grating where these are available. The grating spectrophotometer of E. L. Nichols does not require this correction, but such instruments are not widely serviceable on account of the faintness of the spectra and the overlapping of spectra of different orders.

Spectra are varied to produce a match by means of a rotating sector, a pair of nicol prisms or by varying slit widths. In using variable slit widths, corrections must be applied when the slope of the spectral luminosity curve is steep.

Sensibility should be below 2 percent and systematic errors below 1/2 percent in good spectrophotometry with sufficient intensity of spectra.

Photographic spectrophotometry gives results to about 5 percent or, with extreme care, somewhat better. Most brands of plates are fairly uniform in sensibility (so far as known) from the blue at 0.5μ through the visible violet and ultra-violet out to 0.18 where air itself becomes opaque, but from 0.5 toward red 0.7μ, some stained plates ("spectrum" and "panchromatic" for example) are fairly uniform in sensibility, while others vary greatly with wave length. By taking a series of exposures on the same plate, intensity determinations may be interpolated with some precision. Sensibility to intensity differences is greatest within the range of normal exposure (see Chapter XI.)

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IX.

RADIOMETRY AND SPECTRORADIOMETRY.

Radiation is measured directly as energy in watts or ergs per second by the methods of radiometry. Radiation flux in watts per square centimeter corresponds with luminous flux in lumens, but does not involve the extra-physical factor, Visibility. The geometry of radiation in general, of course, is identical with that of luminous radiation. (See Introduction and Illumination). Since the emission and absorption of radiation is often accompanied by thermal changes, it is sometimes classed with thermal phenomena, but as regards its nature and properties, light in the narrower sense is but a particular class of radiation.

Radiation is specified, whether emitted, propagated in space, or absorbed, when its *intensity*, *quality*, *direction* and *polarization* are given. Intensity and quality are given in watts per square centimeter per unit difference in wave length. Radiation may be studied as a whole or it may be studied a few wave lengths at a time when spread out in a spectrum. Spectroradiometry has been confined largely to the infra-red chiefly because that is the region of most abundant spectral energy. For investigating the visible and ultra-violet regions the most sensitive receivers (radiometers) and the steadiest and most intense sources are required. Spectral energy curves have been obtained for many sources as far into the visible as the green (0.55μ), for the sun and acetylene through the visible out to 0.36μ in the ultra-violet, and for a few intense spark lines to wave lengths as short as 0.19μ.

Radiometry.

The radiometry of total (undispersed) radiation has developed chiefly along two lines, the determination of the

amount of the *solar radiation* (actinometry, pyrheliometry) and radiation *pyrometry*. The diverse forms of radiometers used are of two distinct classes: *absolute* instruments giving results directly in watts or calories and relative instruments or *recorders* whose readings show variations, but are not directly interpreted in watts or calories.

Absolute radiometers are calorimeters absorbing all the radiation passing through a hole, and determine its amount by the heating of water in its double wall. Several have been constructed and used by Abbot[1] and his associates in their work on the solar constant. They are of thin German silver and the size of an ordinary test tube. In use, water flows continuously between the walls and the difference in temperature of the water entering and leaving, determined by thermo-couples, with the measured rate of flow, gives the amount of energy received. Stray radiation is screened off and the interior so designed that none is reflected out. Its readings may be checked by electric heating coils placed within it.

This absolute radiometer is sensitive to differences smaller than 1 calorie per minute (0.00116 watt), but it is difficult to reduce systematic errors due to stray radiation, imperfect absorption and heat losses to that figure. It is too difficult of manipulation for daily service as a recorder, but serves to calibrate and check secondary instruments used as recorders. These cannot be calibrated against a black body or a lamp filament of known energy output and distribution with as small an uncertainty.

In ordinary total radiometers (pyrheliometers) a block, disk or strip of metal is alternately exposed to the radiation and shielded from it, and from its rate of rise and fall in temperature the amount of the radiation is deduced, but with considerable uncertainty on account of the large corrections. The temperature changes are determined with either a mercury or resistance thermometer or with a thermo-couple.

Some early forms of instrument consisted simply of a block or shell of copper with a thermometer inserted. Others

contained water or mercury as heat receivers. The Ångström compensation pyrheliometer contains two similar strips of blackened metal. One of these is exposed to the radiation while the other is heated by a known electric current to the same resistance as the first, thus eliminating the largest corrections. Another modern instrument has as receiver a spiral strip of nickel which serves also as a resistance thermometer. The Callendar instrument is a small thin deep cup to the bottom of which is attached a wire forming a thermo-couple. A balancing current is sent through the junction to cool the receiver (by the Peltier effect) as much as it is warmed by the radiation. Sunshine recorders are arranged to give a continuous record of the temperature of a body heated by solar radiation.

Total radiometry has been applied to *pyrometry* by Féry[2]. A real image of a distant heated body is thrown on a small thermo-junction, the temperature of the body being determined from its heating effect on the junction. Heat losses within the instrument and selective atmospheric absorption prevent its being used as an absolute radiometer in accordance with Stefan's law, but it has proven serviceable as an empirical instrument.

Spectroradiometry.

By determining radiation intensities a few wave lengths at a time with some form of spectroradiometer, spectral energy curves may be constructed and the selective emission, reflection, refraction, and absorption of bodies studied. The region covered is roughly from 0.5 to 100μ, of which the portion from 1.0 to 10μ yields the most interesting results. We shall sketch briefly the most useful instruments and methods used, discuss sensibility and precision, and review a few of the results relating to standards and constants.

Langley was the pioneer in modern methods of spectroradiometry. Twenty years ago he used the mirror spectrometer, rock salt prism, bolometer, and galvanometer in

investigating the solar spectrum essentially as used to-day in the best practice. Paschen took up the study of the properties of various bodies and obtained the approximate form of spectral energy curve of the ideal radiator or black body. He was followed by Aschkinass, Rubens and his associates and Lummer. More recent work has been devoted to strengthening data on the black body, the solar spectrum, atmospheric absorption, the selective properties of various

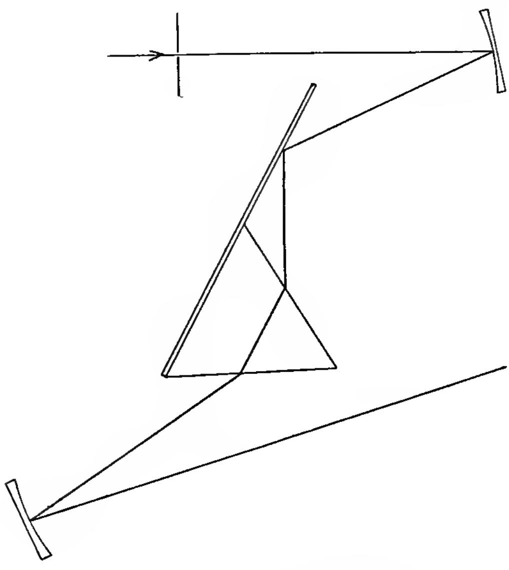

FIG. 57.—Spectroradiometer.

substances, the determination of certain fixed wave lengths, and the extension of the work to still longer and shorter waves.

The spectroradiometer is similar in form to the spectrometer. The ocular and observer's eye are replaced by some form of radiometer, silvered concave mirrors are commonly used instead of the glass objectives of collimator and telescope, and fluorite, rock salt or quartz are used for the prism.

A form of instrument ordinarily used is shown in the figure. The plane mirror attached to the prism, permits both collimator and telescope to remain stationary, different parts of the spectrum being thrown on the receiver by simply turning the prism. This arrangement of plane reflector with prism also preserves minimum deviation after being once adjusted. For work on selective absorption and reflection, in or near the visible region of the spectrum, quartz or even glass objectives and prisms are often used, since the selective transmission of these materials is there too slight to be of any consequence.

The first conditions to be met in spectroradiometry are sufficient intensity and constancy of source, and a receiver of sufficient sensibility and stability. High precision depends largely on a precise knowledge of the dispersion of the prism used, for upon this depends not only the wave length, but the important slit width correction. In the direct determination of spectral energy several other corrections must be accurately known and applied, such as atmospheric absorption, selective absorption and reflection of prism, objectives and receiver, size of ray pencil at the objectives and change of focal length if any.

Radiometers.—Three different types of receivers are of high sensibility and precision and are widely used in spectroradiometry, while many others have been proposed and used to some extent. These three are the bolometer of Langley, the fine wire thermopile of Rubens, and the improved radiometer of E. F. Nichols. The radiomicrometer of Boys was much used in earlier work, but has been displaced by receivers of easier construction and higher stability.

The *bolometer* consists essentially of a short piece of fine, very thin wire whose resistance is changed by the heating effect of radiation to which it is exposed. This bolometer is connected up to form one of the four branches of a Wheatstone bridge so that very minute changes in resistance can be detected. The wire is very fine to make its heat capacity small and very thin so that heat may enter and leave it

quickly; thus as a whole it is sensitive and quickly responsive. The dimensions used are about 10 mm long, 0.2 mm wide, and a few thousandths millimeters thick with a resistance of 1 to 10 ohms. The material used is a metal such as iron or platinum having a high temperature coefficient (0.005 per degree C) of resistance. It is blackened with carbon or platinum black to increase its absorbing power. In the so-called surface bolometer a rectangular grid of thin platinum foil is used instead of the thin wire as a receiver.

Bolometric sensibility rests ultimately with the galvanometer used to indicate when the bridge is out of balance. This must be much more sensitive than any on the market, but may be constructed without difficulty. It is usually made of the Thomson four coil astatic type. The needles are made of small bundles of pieces of hair spring, 1 to 1.5 mm long. The whole system is very light (about 20 mg), and suspended by a quartz fiber. The coils are wound to come very close to the magnet systems, these being the two chief factors in sensibility. A good galvanometer with mirror and scale will respond to currents as small as 10^{-11} ampere with 10 second period and a resistance as low as 20 ohms.

In using the bolometer, it is customary to first balance the bridge with a slide wire, then note deflections on exposure to radiation, sensibility being determined from time to time with a known shunt resistance.

A good bolometric outfit should give a deflection larger than the uncertainty in the deflection (due to drift and unsteadiness) when exposed to a radiation flux of one-millionth (10^{-6}) watt per square centimeter and should measure to within 1 percent 100 times this or 0.0001 watt/cm^2, roughly equivalent to the radiation from an ordinary candle at 1 meter distance. This 0.0001 watt/cm^2 in the yellow-green region of the spectrum is a fairly bright illumination. The eye is of vastly greater sensibility than the most sensitive radiometer.

The bolometer is subject to troublesome variations arising

from thermoelectric effects in the connections, convection currents of air near the strip and various thermal disturbances, giving unsteady deflections and drift of zero. To avoid these the bolometer strip is placed in a massive metal case (sometimes in a vacuum as well) with diaphragms between it and the opening. The bridge arm opposite the exposed strip (sometimes the remaining two as well) is made a duplicate of it to secure better balance and all four arms are placed within the heavy bolometer case to equalize temperatures. The galvanometer troubles most frequently met with are magnetic materials in or on the coils, electrification of the suspended system, vibration of the support, and stray magnetic field.

The bolometer when skillfully constructed and handled, under good laboratory conditions gives higher sensibility and

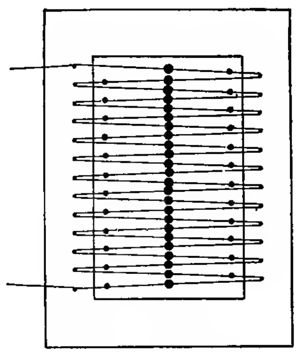

FIG. 58.—Rubens Thermopile.

precision than any other form of radiometer, but it should be chosen only for extended researches where the utmost sensibility and stability are required.

The fine wire multiple *thermoelement of Rubens* is the smallest, simplest and easiest to operate of all the radiometers and has a sensibility roughly 1/10 that of the bolometer. In appearance it is simply a zigzag of fine wire stretched across a rectangular hole in a disk (Fig. 58). Fine

wires of two different metals in short (1 cm) lengths are joined together alternately and strung on a frame so that alternate junctions (10 to 20 of them) are in a line down the center, the other junctions being at either side. When the terminal wires are connected to a sensitive galvanometer and the central line of elements exposed to radiation, the current through the galvanometer indicates the intensity of the radiation.

The metals chosen for the couples are generally iron and constantin or some other pair of metals having large thermoelectric differences. They are fine (0.1 mm) so as to have small heat capacity and conduct heat slowly away from the central junctions, but if too fine, their high resistance affects the sensibility. The central junctions are made large and flat to intercept more radiation and yet thin, to have small heat capacity. The length of the wires must be so great that none of the heat conducted away from the central junctions shall reach those at either side.

The galvanometer used with the multiple element may well be of the type used with bolometers, but a less sensitive and a more stable form is preferable. The limit to the sensibility lies not with the galvanometer, but with the thermoelement and this ends in a compromise between electrical resistance and heat capacity.

A good Rubens element should show little or no drift of the zero. It should reach a definite fixed indication after not over four seconds. The amount of creeping in the final deflection gives the chief uncertainty in the results. Deflections are approximately proportional to radiation flux, but the sensibility must be determined at several points of the scale.

The Rubens element is the best form of radiometer to use where simplicity and convenience are most essential. It may be inserted directly in a telescope or photometer, requiring no connections other than a pair of wires leading to the galvanometer and no other auxiliary apparatus.

The *improved radiometer* developed by E. F. Nichols is a

modification of the old toy called the radiometer constructed so that sunlight falling upon four vanes in a vacuum caused them to rotate. The sensibility of this toy was vastly increased by Nichols by using but two extremely light vanes instead of four heavy ones and by using a delicate quartz fiber suspension instead of a needle.

The vane system used is of about the form shown in the figure. The pair of rectangular vanes are of the thinest mica about 1x4 mm in size and lightly blackened. The whole suspended system weighs but a few milligrams. It is hung in a heavy metal case about 10 cm high and 5 cm in diameter provided with a single re-entrant window of fluorite near the vanes. This window is screened to a narrow slit so that but a narrow beam of radiation falls on one vane. A tiny mirror fixed to the stem permits reading the deflection with telescope and scale.

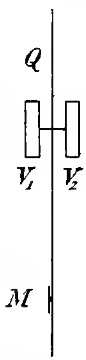

FIG. 59. Nichols Radiometer (Suspension).

The sensibility of this instrument is but little lower than that of the bolometer. The sensibility is greater the less the mass (strictly the moment of inertia) of the suspended system and the finer the quartz fiber suspending it. The reduction of the inertia is limited by the inertia of the mirror that must be used to determine the deflection. Quartz fibers of as small diameter as 0.5μ will just safely support the necessary weight and have been used.

The sensibility is a maximum at about 0.6 mm residual gas pressure in the instrument and falls off rather rapidly toward both higher and lower pressures. The sensibility further depends upon the nearness of the exposed vane to the window. Altogether the sensibility of a well modeled Nichols radiometer may be made nearly equal to that of a good bolometer, but its period is much longer.

The radiometer gives well-defined readings with little creeping at maximum deflection and little shift of zero, but its sensibility is constantly varying due to varying gas

pressure and varying electrical or gravitational attraction between vane and window. It is valued for its simplicity of construction, direct positive action and independence of auxiliary apparatus. It cannot be moved about as freely as the Rubens elements, but does not require the skilled manipulation required by the bolometer.

The Boys *radiomicrometer* is a thermoelement in the form of a small closed loop and suspended between the poles of a permanent magnet. One junction flattened is exposed to radiation. Its rise in temperature causes a current to flow

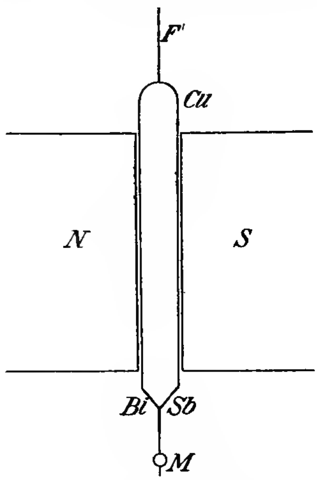

Fig. 60.—Boys radiomicrometer.

around the loop; the loop is rotated by the magnetic field. The instrument is of as high sensibility as any radiometer but is troublesome and uncertain in its readings.

Prisms.—The dispersed spectrum in spectroradiometry is generally secured by the use of 60° prisms of fluorite or rock salt on account of their transparency from the extreme ultra-violet to far into the infra-red. Rock salt may be obtained in large blocks (up to 10 cm) of optical quality, but its surfaces deteriorate rapidly. Fluorite gives good permanent surfaces, but is difficult to obtain in blocks yielding prisms of

over 2 cm face and no prisms have been obtained of greater than 6 cm face.

Glass prisms may be used in the region from 0.35 to 1.0μ. Quartz is transparent from 0.2 to 6.0μ and both quartz and glass prisms are used in and near the visible spectrum. Gratings, both ruled and constructed of fine wire, are ideal as regards uniformity of dispersion, but give such faint spectra that they cannot be used except in the determination of the wave lengths of some intense lines. Silver chloride is extremely free from selective absorption, but has not yet been obtained in large blocks free from turbidity.

For high precision in spectroradiometry it is desirable that the variation with wave length of the refractive index of the prism be as uniform as possible. All of the prism materials mentioned show a high, rapidly changing index in the ultra-violet and a low nearly constant index in the infra-red as the absorption bands are approached. The dispersion curves of a number of materials are given in the table and figure.

REFRACTIVE INDEX OF ROCK SALT IN AIR.

$\lambda(\mu)$	n	Obs.	$\lambda(\mu)$	n	Obs.	$\lambda(\mu)$	n	Obs.
0.18541	1.89384	M	0.88396	1.53401	L	5.8932	1.51601	P
.20447	.76964	M	.97230	.53253	L	5.8932	.51555	L
.29137	.61325	M	.98220	.53243	P	6.4825	.51363	P
.35870	.57932	M	1.03676	.53176	L	6.4825	.51347	L
.44159	.55962	M	1.17860	.53037	P	7.0718	.51106	P
.48615	.55338	M	1.17860	.53037	L	7.6611	.50832	P
0.48615	1.55341	L	1.55514	1.52821	L	7.9558	1.50680	P
.48615	.55340	P	.76800	.52744	P	8.8398	.50203	P
.58902	.54434	L	.76800	.52744	L	10.0184	.49472	P
.58932	.54431	P	2.07352	.52655	L	11.7864	.48182	P
.65630	.54067	P	2.35728	.52586	P	12.9650	.47172	P
0.65630	1.54070	L	2.35728	1.52585	L	14.1436	1.46055	P
.70655	.53863	P	2.9466	.52453	P	14.7330	.45440	P
.76653	.53671	P	3.5359	.52317	P	15.3223	.44749	P
.76824	.53666	M	4.1252	.52165	P	15.9116	.44103	P
.78576	.53614	P	4.1252	.52163	L	20.57	.3735	R–N
.88396	.53401	P	5.0092	.51898	P	22.3	.340	R–N

FLUORITE IN AIR.

$\lambda(\mu)$	n	Obs.	$\lambda(\mu)$	n	Obs.	$\lambda(\mu)$	n	Obs.
0.18560	1.50940	S	1.4733	1.42641	P	3.8306	1.41120	P
.19881	.49629	S	.5715	.42596	P	4.1252	.40855	P
.21441	.48462	S	.6206	.42582	P	4.4199	.40559	P
.22645	.47762	S	.7680	.42507	P	4.7146	.40238	P
.25713	.46476	S	.9153	.42437	P	5.0092	.39898	P
0.32525	1.44987	S	1.9644	1.42413	P	5.3036	1.39529	P
.34555	.44697	S	2.0626	.42359	P	5.5985	.39142	P
.39681	.44214	S	2.1608	.42308	P	5.8932	.38719	P
.48607	.43713	P	2.2100	.42288	P	6.4825	.37819	P
.58930	.43393	P	2.3573	.42199	P	7.0718	.36805	
0.65618	1.43257	S	2.5537	.42088	P	7.6612	1.35680	P
.68671	.43200	S	2.6519	.42016	P	8.2505	1.34444	P
.71836	.43157	S	2.7502	.41971	P	9.4291	.31612	P
.76040	.43101	S	2.9466	.41826	P	51.2	3.47	R–A.
.8840	.42982	P	3.1430	.41707	P	61.1	2.66	R–A.
1.1786	1.42787	P	3.2413	1.41612	P			
1.3756	1.42690	P	3.5359	.41379	P			

QUARTZ IN AIR.

$\lambda(\mu)$	n_0	n_ε	$\lambda(\mu)$	n_0	n_ε	$\lambda(\mu)$	n_0
0.185	1.67582	1.68999	0.396	1.55815	1.56771	2.84	1.5039
.193	.65997	.67343	.410	.55650	.56600	3.18	.4944
.198	.65090	.66397	.486	.54968	.55896	3.63	.4799
.206	.64038	.65300	.598	.54424	.55334	3.96	.4679
.214	.63041	.64264	.656	.54189	.55091	4.20	.4569
0.219	1.62494	1.63698	0.686	1.54099	1.54998	5.0	1.417
.231	.61399	.62560	0.760	.53917	.54811	6.45	.274
.257	.59622	.60712	1.160	.5329	....	7.0	.167
.274	.58752	.59811	1.969	.5216			
.340	.56748	.57738	2.327	.5156			

(L) S. P. Langley, Am. J. Sci., 30, 477, 1885.
(M) F. F. Martens, Ann. Ph., 6, 616, 1901; Ann. Ph., 8, 460, 1902.
(P) F. Paschen, Ann. Ph., 4, 302, 1901; Wied., Ann., 53, 301, 1894.
(R–N) Rubens and Nichols, Wied. Ann., 60, 450, 1897.
(R–A) Rubens and Aschkinass, Wied. Ann., 67, 1894.

The wave lengths of some of the sharpest and best known maxima in the infra-red useful as reference points may be

cited: conducting helium gives the emission lines 0.728, 1.083, 2.058, and 4.054μ. The CS_2 flame gives strong maxima at 7.4 and 8.7μ due to SO_2 as well as the CO_2 band at 4.4μ. Acetylene absorbs strongly at 3.08, 7.73 and 13.63μ, CS_2 at 4.6, 6.8, 11.7, and 13.4μ; mica at 18.4 and 21.25μ. Calcite absorbs strongly at 29.4μ. Rock salt selectively reflects a pair of maxima at 53.6 and 46.4, KCl (sylvin) at

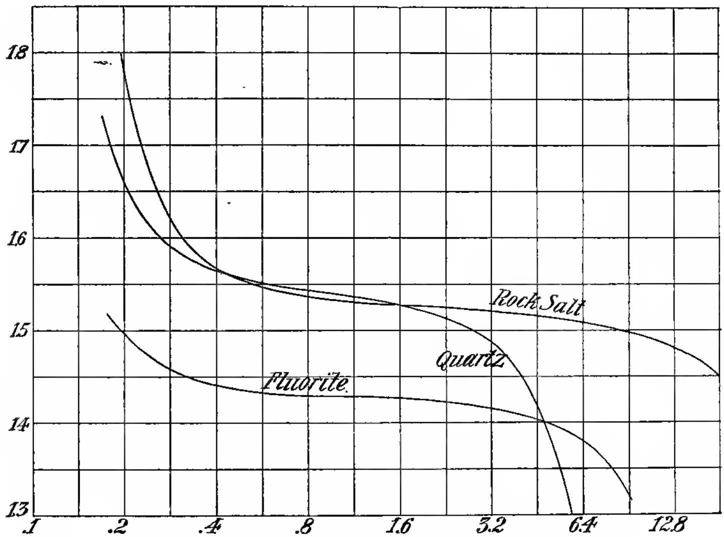

FIG. 61.—Dispersion of Rock Salt, Quartz and Fluorite.

62.0 and 70.3, KBr at 86.5 and 75.6, calcite at 93 and 116μ, the first being the stronger in each case. KI reflects selectively at 96.7μ (nearly 0.1 mm).

The *absorption of the air* on the path of the radiation from source to receiver is practically limited to its water vapor and carbon dioxide content. Each of these absorbs strongly in certain regions and each must be corrected for or eliminated.

The spectrometer from slit to receiver is enclosed in a nearly air-tight case of sheet metal, and within this are

placed a vessel containing dry caustic and another containing P_2O_5. These effectually remove both CO_2 and water vapor. Absorption between source and slit is eliminated by similar means when desired.

Sources.—For all relative work with the spectroradiometer the chief requirements are abundant radiation and constancy over a period of one or two hours. The acetylene flame, Nernst filament, and the electric furnace are all used and may be maintained constant. The energy spectrum of each has been extensively studied.

Precision.—There are three distinct classes of spectroradiometry, each with very different requirements in attaining a similar degree of precision.

When the measurement is of *relative energy* at the same wave length, as in the determination of selective transmission and reflection, readings are taken with and without the body interposed. With sufficiently large, but not too large deflections (5 to 60 cm), such determinations may be made with an uncertainty of 0.1 to 1 percent. The chief source of error is stray radiation of wave lengths different from that for which the spectrometer is set.

In determining *spectral energy curves* or the relative amounts of energy throughout the spectrum, the corrections and sources of error are numerous and difficult. The chief of these are: selective absorption of the blackened receiver, selective atmospheric absorption, selective losses by reflection on the mirrors or lenses, the variable loss by reflection on each face of the prism due to varying inclination and refractive index, and selective absorption in the prism and objective lenses if lenses are used. When a lens spectrometer is used a correction for variation in focal length must also be applied. Finally the energy spectrum obtained must be reduced from prismatic to normal by multiplying by a factor proportional to the dispersion, and lastly the normal spectrum must be further corrected in portions where its slope varies rapidly on account of the finite width of the receiver. Diffuse scattered radiation must be eliminated

so far as possible and the remainder determined by the use of some highly selective absorber.

Of these, the correction for variable reflection on the prism faces may be computed, other corrections are determined experimentally.

A third class of spectroradiometry is the actual determination of the *radiation flux* in watts per square centimeter per unit difference of wave length at a given distance in a given direction from a radiator of given area. This has never been attempted except in some of the work on the full radiator or black body. This work requires not only an application of all of the corrections above enumerated, but a knowledge of the relative energy flux at four points (source, slit, prism and receiver) in the beam of radiation and the sensibility of the receiver in absolute measure (deflection per watt per cm^2). If these are known, a spectral energy curve may be obtained whose integral is the total radiation entering the slit. If the size and position of the slit relative to the source are known, the emission of the source in the direction of the slit may be determined in absolute measure.

Standards and Constants.—Radiation is directly expressible in watts per square centimeter if total radiation, or in watts per square centimeter per unit difference in wave length if spectral radiation and since these are mechanical units, no special radiation unit or primary standard is required. Calibrations are usually made through the electrical units it is true, but this is of no consequence since these units are well established.

However, in many lines of practical work a thoroughly known, reproducible radiator is extremely useful as a reference standard. The service of radiometry to other branches of applied optics lies largely in the study of such reference standards and the determination of the constants of the radiation formulas. No such results that can be regarded as final have yet been attained in radiometry. A few of the more important tentative results are recorded below.

The *sun* is the only natural standard worth consideration

and its radiation varies over a range of 8 percent, while the earth's atmosphere (clear sky) absorbs 20 to 50 percent (mean 0.299) in a continually varying amount. The mean value of the solar radiation is 1.92 calories (15° C) per square centimeter per minute or 0.002234/watt/cm.[2]

The solar spectral energy curve is very irregular on account of the numerous heavy reversed lines. In the following table and figure are given solar radiation[3] at the earth's surface at Washington noon, the mean noon absorption and the radiation without the atmosphere. The energy data is merely relative. All are mean values, 1903–7.

Wave Length.	Solar Energy.	Atmospheric Absorption.	Surface at Washington.
0.385	80	0.410	195
0.390	133	0.460	259
0.394	167	0.499	334
0.404	178	0.553	322
0.415	164	0.575	286
0.442	175	0.631	278
0.458	180	0.647	278
0.476	178	0.674	264
0.486	172	0.689	250
0.497	165	0.702	235
0.523	158	0.717	221
0.537	157	0.725	217
0.570	146	0.745	196
0.610	134	0.768	175
0.633	127	0.791	161
0.661	117	0.815	144
0.728	100	0.850	118
0.769	88.7	0.860	103
0.818	77.9	0.871	89.5
0.877	65.8	0.883	76.5
0.946	54.1	0.892	60.8
1.034	44.6	0.906	48.2
1.127	36.3	0.912	39.8
1.508	18.6	0.923	20.2
2.060	5.2	0.904	5.8
2.428	1.9	0.875	2.2

Of the artificial sources suitable for reference, the full radiator or *black body* is best known. This is the interior of a cavity with opaque walls, the amount and distribution of the radiation being independent of the material of the wall, being a function of wave length and temperature of wall only. In practice such a radiator is approximately realized in the nearly closed porcelain or graphite tube of an electric furnace.

The total radiation flux from such a radiator is (Stefan's Law).

$$5.32 \times 10^{12} T^4$$

in watts per square centimeter with temperature T expressed in centigrade degrees measured from absolute zero ($-273°$ C). Radiation from a body at 100° relative to one at 0° is 0.0731 watt/cm.[2] Each of these constants is still slightly uncertain owing to atmospheric absorption. The Stefan law constant is still further uncertain due to uncertainties in the high temperature scale.

The energy radiated by such a full radiator is distributed throughout the spectrum according to Planck's law,

$$E = C_1 \lambda^{-5} (e^{c_2/\lambda T} - 1)^{-1}$$

in which E is the radiation in watts per cm.^{2} per unit difference of wave length, $C_1 = 3.688$ and $C_2 = 14550$ for λ expressed in microns (μ). The uncertainty in these values is of the order of 1 in 1000.

The *outer surfaces* of heated solids radiate less (5 to 80 per cent.) than a full radiator at the same temperature, the emission varying with the condition of the surface, the substance, and the wave length. Planck's law applies only approximately to such surfaces. The emissivity of polished surfaces is, of course, greater at wave lengths where the reflectivity is low, the emissivity of any surface being the radiation flux density on it relative to that from a full radiator at the same temperature.

At a fixed wave length the relative radiation of any surface at two different temperatures follows closely the Paschen law,

$$\log E_2/E_1 = K(1/T_1 - 1/T_2)$$

The same relation is much used to find the true temperature of a body from its apparent temperature as determined by its brightness with an optical pyrometer.

The emission of the acetylene flame[4] has been determined by several observers whose results are in close agreement in the visible spectrum.

Wave length	0.36	.40	.44	.48	.52	.56	.60	.64	.68	.72
Rel. Emn. Acet.	3.5	5.8	9.7	16.5	27.6	43.7	66.3	96.5	130.1	163.8

The relative emission (color) appears to be nearly independent of atmospheric conditions.

The spectral energy curves of many other radiators have been determined, but not under conditions sufficiently reproducible to permit of their use as reference standards.

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X.

POLARIMETRIC ANALYSIS.

Light more or less polarized in one or several different manners is of frequent occurrence and it is frequently necessary to determine the amount and character of the polarization by some form of polarimetric analysis. Emission, reflection, and transmission, all three produce or alter polarization in general and in many cases a knowledge of the polarization produced by a body gives much useful information as to its nature and properties. The amount of sky polarization has an important bearing on meteorology. The matness of a diffusely reflecting surface is judged by its depolarizing action. The elliptical polarization produced by metallic reflecting surfaces gives the refractive and absorptive indices, while polarization by transmission is of fundamental importance in crystallography, glass testing, chemistry, and saccharimetry. In this chapter are outlined some of the more important methods of detecting and analyzing polarized light.

Detection and Analysis of Polarized Light.

Light is unpolarized when perfectly symmetrical with respect to the direction of propagation and when no mere alteration in relative phase would produce dissymmetry, the latter condition being imposed to exclude circular polarization. Polarized light may be either plane polarized, elliptically, or circularly polarized. Partly polarized light is a mixture of any of these forms of polarized light with unpolarized light in any proportion. Emitted and reflected light, except in special cases, is partly (plane or elliptically) polarized. During transmission through bodies not isotropic

(crystals, active solutions, and bodies in a magnetic field) light is completely polarized, either plane or circularly. After transmission through such bodies, light originally plane polarized may be plane, elliptically, or circularly polarized, depending on the nature and thickness of the body traversed.

Partial Plane Polarization.—Sky light and reflected light in general, is partly plane polarized. The percentage polarization varies (through a maximum) with the angle of reflection and with the matness of the surface. A mat surface or turbid medium tends to depolarize light previously polarized incident upon it. The partial plane polarization caused by reflection is due to the unequal reflection of the wave components parallel and normal to the surface.

Various forms of polariscopes (Arago, Babinet, Bravais, Senarmont, Savart) have been devised for detecting polarization. These consist essentially of a pair of phase wedges with a nicol or tourmalin plate for viewing them. At intervals along the wedges the relative retardation of the two components is such that the viewing nicol or tourmalin cuts out the polarized component, producing dark bands.

The relative intensities of light and dark bands are thus in ratio $(U+P)/U$, total light: unpolarized light. They shade off into one another according to the sine square law. The *percentage* polarization is determined from the relative intensity of light and dark bands or by balancing up with a known amount of polarization from a reflecting plate. The *azimuth* of the polarization is determined by rotating the polariscope to the position of maximum contrast or better, by locating the two positions on either side where no bands are visible. By noting whether the central band is light or dark and checking against a plane surface (the plane of polarization lies in the plane of incidence) the direction of the polarization may be determined.

Sensibility in detecting plane polarization is fixed by the ability to detect the alternating light and dark bands. Hence if photometric sensibility is say 2 percent, the least percepti-

ble polarization is also about 2 percent of the whole. This is also the uncertainty in the measurement of the percentage polarization; whether this is balanced up by a known polarization or the relative intensity of light and dark bands is directly determined with a photometer. If the bands are photographed, the sensibility is hardly as great (2 to 5 percent) as when observed visually.

In determining the azimuth of the polarization by locating two positions in which bands disappear the sensibility of setting is again limited by the photometric sensibility of the eye or photographic plate. This corresponds to an uncertainty of about 1 degree for light completely polarized and a proportionately larger uncertainty as the percentage polarization is less. If but 5 or 10 percent of the light is polarized, better azimuth settings can be made on the position of maximum contrast than on the position of disappearance of bands.

Plane Polarization.—With light known to be completely plane polarized, it is necessary in certain work (*e.g.*, in rotimetry) to locate the azimuth of the plane of polarization with the utmost precision. The simple direct method of doing this is to observe the beam through an ordinary nicol (simple analyzer) setting for extinction. In the other method, two parts of the visual field are brought to equality of brightness. The double field is obtained by some sort of compound analyzer, two nicols mounted at a slight angle, a single nicol with a flake each of right and left rotating quartz on the front face or some similar device.

Sensibility.—With a simple analyzer, any position of the nicol at which no transmitted light can be seen is a setting. Hence the maximum error in a setting is an angle such that the transmitted light is below the threshold of vision L_0. Then if the light used is sensibly monochromatic

$$T = L \sin^2 U < L_0, \quad U > (L_0 : L)$$

T is the intensity of the transmitted light, L its maximum value, and U the maximum error in setting. L_0 is of the

order of 0.0005 meter candle so that L would have to be about 50,000 m.c. to bring U down to 0.01 degree[1].

With a compound analyzer the sensibility depends on the angular difference between the two parts of the visual field (analyzing angle) as well as on the photometric sensibility of the eye. The maximum error in setting has been shown to be

$$U = 1/8\ PA$$

where P is the photometric sensibility of the eye (2 to 10 percent, cf. Chapter V) and A is the analyzing angle. With the compound analyzer the intensity of source has little effect on P so long as it is not too low, but does affect the permissible analyzing angle A directly. With $A = 2°$ and $P = 0.04$, $U = 0.01°$, provided the source used be sufficiently intense.

Comparing the sensibility (U_2) of the compound with that of the simple analyzer

$$\frac{U_2}{U} = \frac{P}{4}\left(\frac{L_1}{L_0}\right)^{\frac{1}{2}}$$

where L_1 is the illumination of the field of the compound analyzer at a setting. If $L_1/L_0 = 100$ and $P = 0.04$, then $U_2/U_1 = 0.1$ or the compound analyzer is ten times as sensitive as the simple nicol.

In practice the simple nicol is very little used as an analyzer in precision work. Compound analyzers with angles of from 2 to 10 degrees are mostly used, the analyzing angle depending on brightness of the sources of light and material available. In one recent instrument the analyzing angle may be varied[2] continuously during use as desired.

Lippich[3] constructed a rotimeter in which the Landolt band (seen when an intense source is viewed through a pair of single nicols exactly crossed) is brought to bisection by a central cross wire. With this analyzer settings may be made to 2 or 3 seconds, but excessively intense sources are required and the method is little used.

Elliptical Polarization.—The analysis of elliptically polarized light consists of two distinct steps, first, the determina-

tion of the azimuth of one of the axes of the wave ellipse, and secondly, of either the ratio of the two axes or the phase lag. There are two different methods of determining these quantities. The first method is to reduce the elliptical to plane polarization with a quarter wave plate (mica or quartz) and then observe with a nicol the two azimuths (of quarter wave plate and nicol) with which extinction may be obtained. The azimuth of the wave plate gives the position of the elliptic axes, the position of the nicol the ratio of those axes.

The second method is to use a wave plate of variable thickness (compensator) and vary this thickness until extinction can be obtained with an analyzing nicol. The position of the nicol gives the ratio of axes while the phase difference is computed from the thickness of compensator plate. Variable thickness is secured by the use of a pair of quartz wedges, both cut parallel to the axes, but with axes at right angles (Babinet form) to each other. The variable thickness is obtained by sliding the wedges past each other, and compensation is obtained in narrow bands. In the Soleil form of compensator a pair of wedges cut alike are combined with a plate cut with axis at right angles to the axis in the wedges, so that the whole field is of a uniform shade.

Sensibility.—This double setting of both phase plate and nicol to secure extinction is, of course, not capable of very high precision. Errors as large as 2 degrees are to be expected. Many attempts to increase the sensibility have been made. Replacing the simple nicol by a compound analyzer has given increased sensibility as in rotimetry. The same half-shade principle has been applied to the phase plates. A thin mica strip added, gives over compensation in one half the field, and under compensation in the other, that is the elliptical polarization is a little more than reduced to plane in part of the field and a little less in the remainder. With this arrangement and double nicol the visual field is in quarters and at a setting the four quarters appear equally

bright. The uncertainty in setting has been reduced to as low as 1 percent by this means.

Properties of Materials.

Various materials affect the state of polarization of transmitted or reflected light and from the change produced it is frequently possible to deduce the nature or properties of the material.

Double Refraction.—Double refraction in bodies ordinarily isotropic indicates a state of strain; in certain crystalline bodies it is a means of determining useful structural properties. The simple test for the existence of double refraction is to place the object between crossed nicols and observe whether the field is lightened as a whole or in part. Feeble double refraction may escape detection in this test, however, even with strong illumination of the polarizing nicol. This method is extensively used in testing optical glasses for strain, high sensibility being unnecessary. In some optical works, the specimen is placed between large crossed nicols rotating in unison, the rotation eliminating those interference figures not due to strain in the glass.

A more sensitive test suitable for very *feeble* birefraction is to first place between the nicols a sensitive plate, either gypsum giving first order red, or a quartz plate of sensitive tint. The introduction of a specimen having a slight trace of birefraction will change the tint of the field. It is difficult to find a specimen of blown glass which does not show birefraction under this test.

Pleochroism.—Dichroism and trichroism are as a rule easily apparent to the unassisted eye. The relative absorption in the different directions is determined by spectrophotometry. A simple test is to place the specimen on a pair of steep phase wedges. Bands will appear if there is but a trace of pleochroism.

The sensibility is that of ordinary spectrophotometry, but the precision attainable depends largely on the specimen

available. The correction for reflection requires a knowledge of the refractive indices.

Birefractometry.—The refractive indices of birefracting media may be determined by deviation in a prism, by total reflection, by phase retardation in plates and by other methods. The method of total reflection is the easiest and simplest, requiring no computation nor any knowledge of crystal form or orientation of the face used. For liquids (in a magnetic field) the retardation method must of course be used.

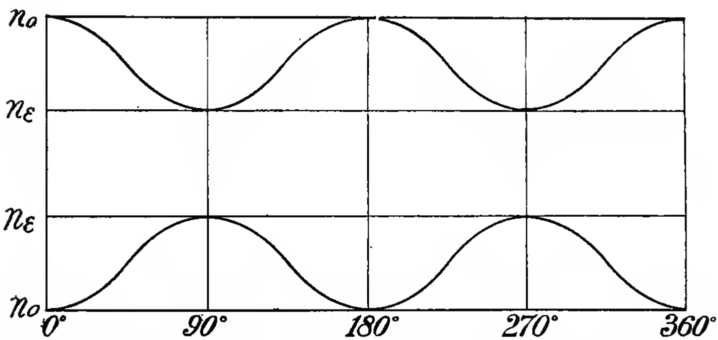

FIG. 62.—Uniaxial crystal, face parallel to the axis, negative above, positive below.

If a plane crystal face, natural or prepared, be placed on the plane of a refractometer of the total reflection type, the visual field will in general show two and but two readings of the refractive index. On rotating the specimen in its plane these indices will in general vary, and from the maximum and minimum values the principal indices are obtained.

(1) If both readings are constant the crystal is uniaxial. The face is normal to its axis and the readings the principal indices. (2) If one reading is constant and the other varies, the specimen may be either uniaxial with face oblique to the axis or biaxial with face one of the three principal sections. Turn a second face of the specimen to the refractometer.

If one reading is still constant and the other gives the same maximum (or minimum) as the first face, then the crystal is uniaxial. The constant index is the ordinary index, and that value of the other index most different from the ordinary is the true extraordinary index, lower than the ordinary index for a positive crystal, higher for a negative.

(3) If the faces applied to the refractometer are those of a biaxial crystal, the highest and lowest readings on the two faces will agree, while of the two intermediate ones, one will be the same on the two faces, while the other will not. The three which are the same on the two faces are the three principal indices of the crystal $n_1 n_2 n_3$.

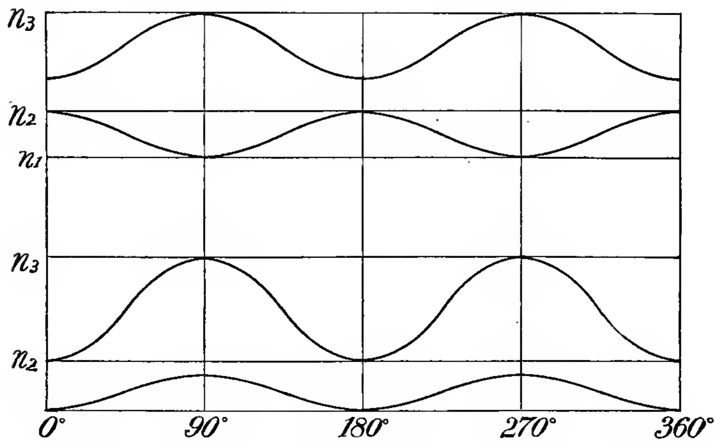

FIG. 63.—Biaxial crystal, index observations on two different faces.

In the figures are shown specimen readings with the crystal face inclined at different azimuths with respect to the plane of incidence. Figure 62 shows typical readings on a face of uniaxial crystal, positive above, negative below. Figure 63 shows similar series of readings on two faces of a biaxial crystal, the three true indices being indicated.

The use of these constants in locating the optic axes and wave ellipsoids is described in works on crystal optics. The

relations between the optical constants of crystals and their geometric forms and space relations, determined with the polarizing microscope, must be referred to special treatises on crystal optics.

Rotation of the Plane of Polarization.—The three classes of chiral properties of bodies (see Introduction) are in many cases extremely useful for identification and even for estimation of quantity. Structural rotation frequently (but not always) occurs with ordinary double refraction in solids. The molecular rotation of fluids apparently has no relation to ordinary birefraction. In all three kinds of rotation; structural, molecular, and magnetic, the rotation is due to circular birefraction, but is not conveniently measured as such.

Tests for slight rotation are made between crossed nicols in parallel light with a double pair of rotation wedges. The design of F. E. Wright is probably the most sensitive of any. Rotation caused by a specimen is shown by a relative displacement of the rotation bands.

Rotimetry.—The rotation produced by a body is determined by sending plane polarized light through it and measuring the difference in the azimuth of the polarization as above. The *sensibility* is that of the half shade analyzer with analyzing angle adjusted to the intensity of the transmitted light, roughly 0.01 degree. The specific rotation of a substance is of course determined most accurately when the total rotation is conveniently large.

The greatest difficulty in the determination of rotation is rotary dispersion. This necessitates (in general) the use of monochromatic light, and monochromatic sources are difficult to obtain of sufficient intensity and of the required wave length. The sodium arc and oxyhydrogen flame and the mercury arc (green line) are used extensively.

In saccharimetry, rotimetry is determined by a null method, using white light. The rotary dispersion of quartz and of sucrose being nearly the same, the rotation caused by the 20 cm sugar solution is compensated by a movable wedge of quartz, the analyzer remaining fixed, so that intense

white light may be used for illumination. The sensibility is thus that of the half shade analyzer in the absence of rotary dispersion.

Rotation and Retardation.—Metallic reflection affects both the azimuth and the phase of incident plane polarized light. From the amount of the rotation and phase retardation, the refractive and absorptive indices of the metal are computed. Sensibility and precision depend largely upon how accurately elliptically polarized light can be analyzed (see above).

Many methods have been used in the study of metallic reflection. The oldest method was to use plane polarized light at an azimuth of 45 degrees and incident at any convenient angle on the mirror. Another method is to use circularly polarized incident light and vary the angle of incidence until the reflected light is plane polarized. Voigt and Minor[5] developed a very ingenious photographic method applicable far into the ultra-violet as well as to the visible spectrum. A pair of phase wedges and a pair of rotation wedges introduced in the path of the reflected beam before the simple nicol analyzer produce on the photographic plate a rectangular system of points. The displacement of these points by the reflection gives change of phase along one coordinate, and rotation along the other.

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XI.

PLATE GRAIN AND SENSITOMETRY.

The photographic plate and the human eye stand in a very similar relation to optical instruments and optical measurements, being in many cases supplementary or even interchangeable. Either plate grain or retinal grain set a limit to the resolution required in an image. Photometric and chromatic sensibility correspond closely; but with the difference that the plate action is cumulative while retinal action is not. The photographic effect depends chiefly upon the total *energy* received and but slightly upon the *rate* at which it is received; with the retina the reverse is true.

The purely physical properties of the photographic plate are here outlined, the size and distribution of the grains in relation to resolving power and the degree of blackening as dependent upon the intensity and wave length of the incident radiation, time of exposure and treatment of the plate.

Grain.

The coating of an exposed and developed plate (or film) contains numerous minute grains of metallic silver imbedded in the transparent gelatine, the blackening being due entirely to the silver grains. These grains are just too small to be seen by the unaided eye, but with an enlargement of 10 diameters they may be detected and with a hand magnifier of 10 mm focus giving a magnification of 25 diameters they may be distinctly seen. With a microscope of moderate power (200 to 500 diameters, N.A. = 0.5), their size, form, and distribution may be definitely determined.

The silver grains are angular polygons in plan with three to five straight sides and varying in form from thin wedge-shaped triangles to nearly regular pentagons. Their mean

form varies somewhat with the method of manufacture. They occur chiefly singly, though an occasional clot of 3 to 10 or more grains is found in all but the best plates. The proportion of clots is generally greater in high speed plates than in the slow lantern and process plates, and much greater in poor plates than in high grade plates. In the best grades, few clots are found in even the higher speeds. Less than 5 percent of the grains should be in clots for any work in which fine details are of importance. The amount of this clustering of grains appears to be independent of the exposure and but slightly affected by the kind of (good ordinary) development, but prolonged development increases their number and certain solutions used in the after treatment of plates are extremely effective in forming clots.

The gelatine film containing the grains is ordinarily 15 to 18μ (0.015 to 0.018 mm) thick and the grains fairly uniformly distributed through it. An extra thin film is 6 to 9μ thick, while a double-coated plate has a film as thick as 30μ. If the period of setting after coating the plate is not properly adjusted to the quality of the coating the distribution may be far from uniform. The total number of silver grains present is roughly two or three times the number necessary to render the film opaque if all were reduced to metallic silver.

Individual grains are from 1 to 3μ (0.001 to 0.003 mm) across in nearly all ordinary and higher grade high speed plates, but in contrast plates (lantern slide and process plates) they are considerably finer, 0.5 to 1.5μ. Size of grain is independent of exposure and varies but slightly with any normal standard developer. With prolonged development, the grains tend to increase slightly in size by accretion.

In wet plates the grains appear under low magnification to be like those of the best dry plates, 0.5 to 0.6μ in diameter and quite uniform in size. But under the highest power (oil immersion) each of these grains is seen to be an aggregate of 4 to 10 finer particles, usually separated by about their own diameter (0.15μ) and very uniform in size.

Resolving Power.—The so-called resolving power of photographic plates is determined by the distance by which two images must be separated in order to appear distinct from each other. If a carefully prepared straight edge, like a razor blade, be placed in contact with the film of a plate and a print taken in parallel light, the developed image will show under a microscope a distribution of grains about as shown in the figure. The ordinates indicate the number of grains per unit area present. The dotted ordinate indicates the exact position of the edge of the blade.

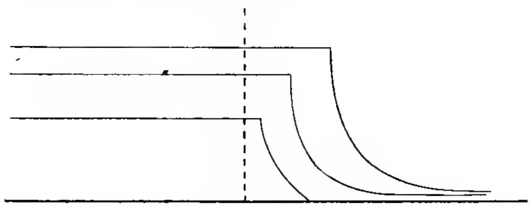

FIG. 64.—Number of grains per unit area at edge of a shadow.

There is a well marked region of shading off between the high and lower level, the width of which is large in comparison with size of grain. This region of diffusion is 20 to 30μ wide in all ordinary plates and is but little affected[1] by exposure or development or even by the action of intensifiers and reducing baths. It is practically absent from the special plates with transparent films.

It is the width and shape of this shading off which determines resolving power. Suppose there is a second image facing the first with a distribution of grains as shown by the dotted curve. The resultant number of grains will show a slight depression above the point of intersection if at that point the curves are less than half their full height.

This test for plate resolution cannot safely be made with an image formed by a lens. The resolving power of the lens ($F/10$ say), residual aberrations and the difficulty in focussing all may introduce errors nearly as large as the effect sought.

Irradiation.—If a straight edge and plate be exposed as above but for times proportional to 1, 2, 4, 8, etc., the edge of the shadow will retreat according[2] to the law,

$$d - d_0 :: \log T$$

The distribution of the grains is about as shown graphically in Fig. 65. This spreading of the image appears to be due simply to the light diffusely reflected in the film and to that regularly reflected from the back of the plate. In transparent films the effect is very slight.

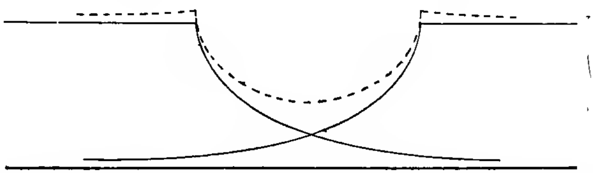

FIG. 65.—Typical distribution of plate grain, illustrating irradiation and resolving power.

Few definite results have yet been obtained in the many important problems relating to plate grain, but plate manufacturers are giving these problems increasing attention.

Sensitometry.

An exposed and developed plate will transmit at every point a certain percentage of the incident light. This percentage transmission corrected for the slight losses by absorption and reflection in the entirely unexposed portions of the plate is the *transparency* of the plate. In other words, transparency is the relative transmission of exposed and unexposed portions of the plate. *Opacity* (O) is the reciprocal of transparency. *Density* (D) is the (common) logarithm of the opacity

$$D = \log_{10} O = \log_{10} \frac{I}{I_0}$$

Opacity varies from 1 (clear plate) to ∞. Density varies from *o* (clear plate) to ∞. It is a measure of the amount of silver reduced in the film. Unit density is the density of a deposit which transmits 1/10 as much light as the unexposed clear plate.

Density depends upon the *number*, *size*, and *distribution* of the reduced grains in the coating. The precise relations have never been worked out, but are no doubt in accordance with the laws of probability except perhaps as regards distribution in the film normal to its face. In the best modern brands of plates both size of grain and distribution within the coating are much more uniform than in older and cheaper brands.

The Characteristic Curve.—If a plate be exposed for times proportional to 1, 2, 4, 8, 16, . . . and developed, the densities in the series of exposures will show a fairly uniform but not exact proportionality to *log t*. As shown in Fig. 66 the curve of density (=log opacity) plotted against log (time of exposure) rises slowly at first then rapidly and steadily, then more slowly. With long continued exposures the density falls again (reversal).

The *length*, *position* and *direction* of the central straight portion of the curve is characteristic of the plate and from these data certain constants may be derived that are independent of developer and development. These relations were discovered by Hurter and Driffield in their classical investigation of plate action and upon them are based the whole of sensitometry.

If the straight central portion of the characteristic curve be projected backward to the time axis, the intercept on this axis is the *inertia* (*i*) of the plate. The reciprocal of the inertia is proportional to the *speed* of the plate, the constant of proportionality being a matter of choice. Eight or ten different speed factors are in use by different persons and firms. Hurter and Driffield use 34. Using the number 50, the observed speeds are approximately: for lantern plates 1, process plates 5, ordinary plates 12 to 60, rapid and extra

rapid 100 to 200 and for the highest speeds yet produced about 400.

The speed number obtained depends upon the time unit employed and the intensity and quality of the light to which the plate is exposed, but does not depend upon the developer or development for any fresh ordinary standard developer.

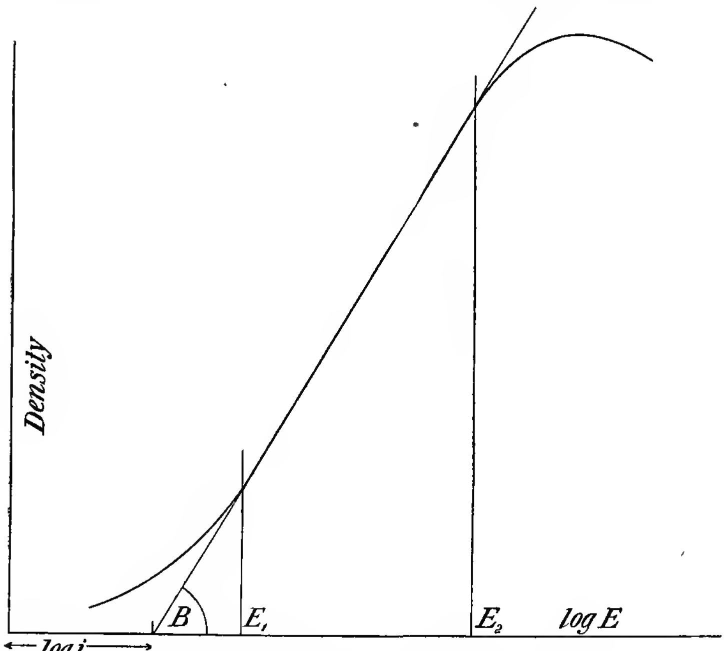

FIG. 66.—Characteristic curve of a plate.

The unit of time chosen is the second, of intensity the meter candle. The quality should be spectral (solar) white, not merely sensation white. Screened acetylene is used by the best workers, but even this departs considerably from spectral white even with the most carefully selected blue screens.

The length of the straight portion of the characteristic curve gives the *latitude* of the plate. If E_1 and E_2 are the

exposures corresponding to the limits of the straight portion, then E_2/E_1 is the latitude. Within this range of exposures from E_1 to E_2, contrast is independent of exposure. Other things being equal, E_1 is the limit of *under exposure* and E_2 of *over exposure*. Other things being equal, under exposures give greater and over exposures less contrast than that of the object photographed, while *normal exposures* give the same contrast as the original. The latitude of a good ordinary plate is in the neighborhood of 16.

The slope of the straight portion of the characteristic curve is steeper the longer the time of development, but approaches a fixed finite value for very long development as shown in Fig. 67. The tangent of the angle between this

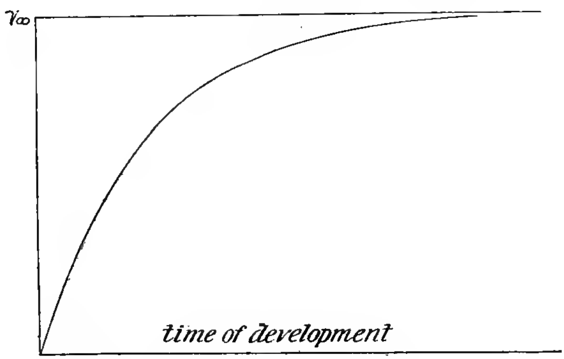

FIG. 67.—Development factor γ as a function of time of development.

portion of the curve and the time axis is called γ, the *development factor*. If two exposures E_1 and E_2 are made on this portion of the curve, and the corresponding densities D_1 and D_2 determined, then γ may be found from

$$\gamma = \frac{D_2 - D_1}{\log E_2 - \log E_1}$$

For *normal development*, $\gamma = 1$ and the negative presents contrasts the exact inverse of the original. When $\gamma > 1$

contrasts are increased, while for $\gamma < 1$ they are decreased in comparison with those of the object photographed.

The development of a photographic plate follows the ordinary physico-chemical reaction law

$$Kt = \log_e \frac{\gamma_\infty}{\gamma_\infty - \gamma}$$

or

$$\gamma = \gamma_\infty (1 - e^{-kt})$$

the equation of the curve in Fig. 67, t being the time of development and k a constant called the *velocity constant* of development.

The value of the velocity constant k varies a great deal with the brand of plate and with the developer used. With an ordinary standard developer at 20°, k is of the order of 0.2 to 0.4 for slow plates and 0.05 to 0.2 for fast plates. With ordinary development, $\gamma = 1$ to 1.4 (normal to slightly enhanced contrast) and γ_∞ varies from about unity (low value) up to 3.0 or over for very high values.

The values of γ_∞ and k together determine the developing properties of a plate. With:

γ_∞ high and k high the image appears quickly and gains rapidly in density.

γ_∞ low and k high the image appears quickly, but does not gain density.

γ_∞ high and k low the image appears slowly, but gains rapidly in density.

γ_∞ low and k low the image appears slowly and fails to gain density.

The variation of plate speed with wave length (*chromatic sensibility*) is an important factor in their properties, but has never yet been determined for any plate for lack of a suitable source whose energy values were known through the visible and ultra-violet regions. Plates are known to be as sensitive in the ultra-violet as in the visible as nearly as we can tell by rough estimates of relative energy. The chro-

matic sensibility of all ordinary plates is not very different from the curves shown in Fig. 68.

It is sensibly uniform throughout the ultra-violet and down through the violet and blue of the visible spectrum to about wave length 0.5μ. From that point to the red, plates differ widely in sensibility. Unsensitized plates have very low sensibility in the green with a slight maximum in the yellow orange. The older sensitized plates ("isochromatic") have greatly increased sensibility in the green and yellow.

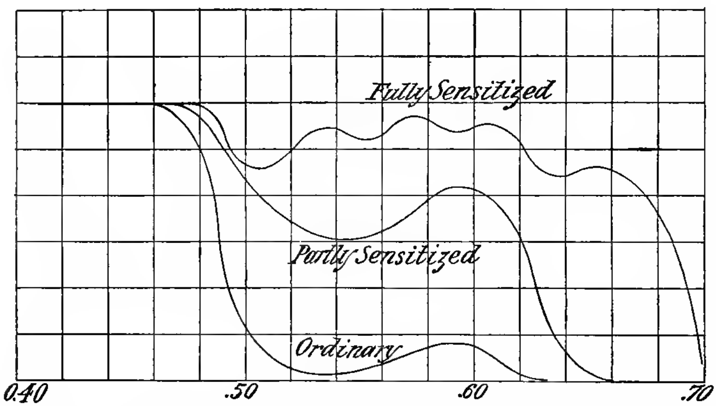

FIG. 68.—Approximate relative wave length sensibility of three types of plates.

The newer "panchromatic" and "spectrum" plates show but a slightly decreasing sensibility out to 0.65 or 0.7 in the red.

Rate of Exposure.—If two exposures are made, one to light half as intense as the other, the second exposure will have to be a little more than twice the first to produce equal density. The relation

$$It^{\kappa} = constant$$

has been found to hold in every case where intensity I and time of exposure t alone are varied. This is known as Schwarzschild's law. The exponent κ is found to lie between 0.7 and 0.95 for various brands of plates. With prolonged development at higher temperatures, it falls

slightly in value. It is also a function of density of image and of wave length. It may be noted that the effect is in the nature of a fatigue lag.

Temperature and Humidity.—Plates show a small but definite variation in speed with temperature. Some plates at some temperatures increase in sensibility with rise of temperature, others decrease. Many brands of plates have a maximum speed at a certain temperature.

Some tests of the effect of *humidity* show a decrease in sensibility with increase of humidity.

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XII.

INTERFEROMETRY.

The shortness and reproducibility of wave length of light waves make them valuable tools in precise determinations of position and displacement by interferometry. Light waves are roughly half a micron ($0.5\mu = 0.0005$ mm) in length, a little larger than the smallest objects resolvable (*e.g.*, fine plate grains) under a high power microscope. Those light waves coming from luminous gases (spectrum lines) appear to be given off in trains of 50,000 to 300,000 waves each, *i.e.*, 1 to 6 inches (25 to 150 mm) in length, while the waves composing the continuous spectra from heated solids are thought to be but single pulses.

Conditions for Interference.—To produce interference the waves brought together must be (1) either parts of the same original wave or pulse or at least from the same wave train. The farther apart in the wave train the combining waves are taken, the less perfect the interference, this distance apart being determined by path difference.

(2) Interference is best when the waves are brought together at a small angle and decreases to zero when the waves come together at right angles to each other. (3) If plane polarized, the two waves interfere best when polarized in the same plane and not at all when polarized in normal planes.

Under suitable conditions then, light waves may be divided and reunited after traveling paths of different lengths or suffering different changes of phase or both. Where the path difference (or phase difference) is half a wave length, no light appears. When this difference is zero the two sets of waves combine in full strength.

Tests of Surfaces.—Two surfaces are tested for parallelism by diffuse illumination with homogeneous light and inspec-

tion of the resulting interference pattern. A source S (Fig. 69) is mounted behind a ground glass screen A elevated for comfortable viewing. The source may be a sodium flame or a Plücker tube of helium or mercury vapor. The optical surfaces L_1L_2 are carefully freed from dust with a camel hair brush and placed together. If the fringes are fine and straight, the surfaces are not in good contact. If several mm broad, the contact is good and if circular the surfaces are parallel.

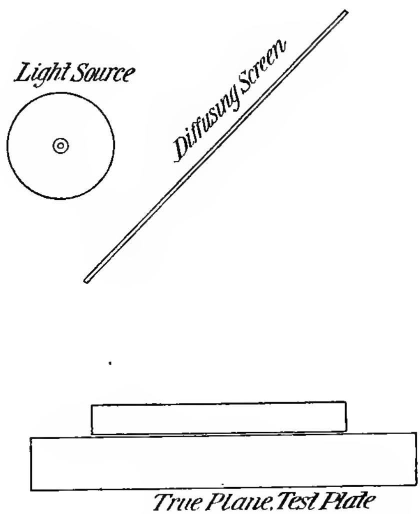

FIG. 69.—Apparatus for testing plane surfaces.

Irregularities in the forms of the fringes indicate corresponding irregularities in one or both surfaces; which surface is defective is determined by moving one surface relative to the other. The sensibility of the method is 1/10 to 1/20 of a wave length (0.05 to 0.025μ). Optical shops keep carefully prepared *true planes* against which to test other surfaces.

Tests for Slight Displacements.—With two surfaces arranged as above to show interference, if one is moved normal to itself the fringe system will also move in the direction

normal to itself, a displacement of half a wave length in the surface giving a displacement of one whole band.

If it is desired to determine a slight displacement like that of a clock pier or the end of a heated bar or tape, plane mirrors are used and parallel monochromatic light. Two different arrangements are used as shown. The simpler is to mount a silvered mirror on the object and close in front and parallel to it a half silvered (semi-transparent) mirror, the light being brought in by a condensing lens and the unsilvered mirrors through which the fringes are viewed.

The second arrangement is essentially the Michelson form of interferometer. The incident light is split into two beams by the half silvered diagonal mirror, one beam going to a fixed mirror and back, and the other to the movable mirror and back to the observer.

With either arrangement displacements as small as 0.01λ may be detected and estimated. Large displacements can be followed and estimated only if they occur slowly.

Calibration of Short Lengths.—Short distances up to about 1 mm may be conveniently determined directly in wave lengths with an interferometer of either the Fabry-Perot or Michelson type. The method is particularly applicable to screw calibration and short distances where the labor of wave counting is not so great. One mirror remains fixed while the other moves with the reading microscope or screw nut. The monochromatic illumination used may be any of the prominent spectrum lines of cadmium, mercury, helium, neon, etc., whose precise wave-length is known in terms of the meter. The narrower lines of cadmium or neon are preferable to the broader complex lines of mercury or helium, but the latter are much more readily produced.

To avoid the labor of counting waves (about 2000 per millimeter) over the larger distances, light of two or even three different wave lengths are used together. From the known ratios of these wave lengths and the observed coincidences of the interference bands, the total displacement is estimated directly. The ratio of wave lengths should be moderate,

say 9 : 10. If too nearly equal, the coincidences are too ill-defined, if too widely different they are too frequent.

The *sensibility* and *precision* attainable depend largely upon the careful adjustment of the mirrors and the homogeneity and intensity of illumination. The maximum sensibility, 0.05 to 0.02μ is roughly equal to that of setting a high power reading microscope on a good line. The inter-

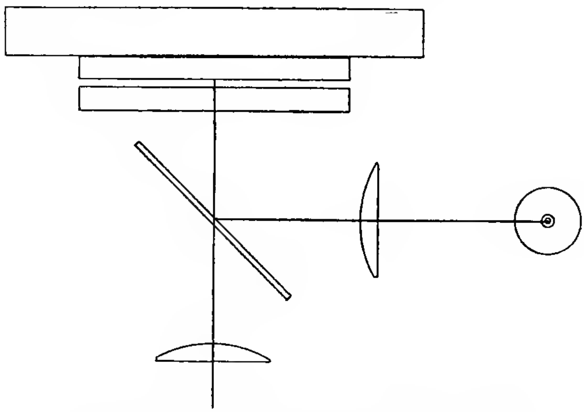

FIG. 70.—Apparatus for observing slight displacements.

ference method is free from the systematic focal and screw errors of the reading microscope, but may introduce errors due to defective sliding ways.

Comparison of Wave Length with Meter.—The light wave furnishes an ideal length standard in that it is easily reproducible from specifications, while the international standard meter can only be reproduced by copying. It is hence highly desirable to know the relative lengths of the meter and the length of some suitable light wave. Michelson[1] and Morley (1889) devised plans for a direct comparison of light waves with the meter. In 1893, Michelson,[2] assisted by Wadsworth, Benoit, Chappuis and Guillaume, successfully compared one of the prototoype meters in Paris with the wave lengths of the red, green, and blue cadmium lines from

a Plücker tube. The values obtained for these wave lengths in dry air at 15° and 760 mm are

Red, $643.84696\mu\mu$,
Green, 508.58219,
Blue, 479.99087.

The comparison has since been repeated by several different observers by slightly modified methods with the same results to the last significant figure.

The method in brief was to count the number of cadmium waves over a short interval and then to step off the meter in terms of this interval, using white light and four mirrors The four square mirrors were disposed, two on each of two massive blocks, one mirror being above and behind the other on the same block. With the two blocks side by side, the four mirrors in elevation formed the four quarters of the square.

The three chief sources of error are (1) in setting on the defining lines of the meter bar with the reading microscope, (2) losing count of a whole cadmium wave, and (3) an error of a small fraction of a wave in stepping off with white light. The first errors with care are less than $\lambda/10$ (about 0.05μ). The second are eliminated, and the third should be well under $\lambda/10$. Temperature, barometric pressure and humidity corrections need be made with but ordinary precision, since they affect but $(n-1)$ while the wave lengths are inversely proportional to the index n itself. The total uncertainty may then well be under 1 in ten million.

Intercomparison of Wave Lengths.—Interference methods are particularly well adapted to determinations of relative wave lengths. The methods used are simple in theory and of very considerably higher precision than any others. By referring back to the cadmium lines, the wave length of any spectrum line that is sufficiently homogeneous may be found in terms of the international standard meter.

The two interference methods of highest precision each make use of a pair of parallel plane mirrors. The distance

between the mirrors is a whole number of waves plus a fraction of a wave. This fraction is determined either from the diameter of an interference ring (method of diameters) or from a slight known flexure of one mirror support (method of flexure) to just compensate for the wave fraction.

In using the *method of diameters*, the two mirrors are lightly silvered and mounted a fixed distance apart with three small invar balls between. Final adjustment to parallelism is made by varying the pressure. An image of the ring system is formed by a small mirror or well corrected lens, and the image either measured directly with a micrometer ocular or photographed and then measured.

If known and unknown wave lengths (λ_0 and λ) correspond to diameters D_0 and D, and F is the equivalent focal length of the lens or mirror used to project the fringes then,

$$\frac{\lambda}{\lambda_0} = \frac{P_0}{P}\left(1 + \frac{D_0^2 - D^2}{8F^2}\right)$$

where P is the order of interference of the ring observed, twice the number of whole waves between the mirrors minus the number from the center out to the one observed. The precision of the method depends upon the precision with which D and F can be measured. With care, the uncertainty in relative wave length may be as low as 1 : 1,000,000.

The method of flexure compensation is simpler and less liable to systematic error. The back mirror is full silvered, the front mirror barely transparent and mounted on the center of a stiff steel bar about 6x12 mm. in section, displaceable by a light spring. The extension necessary to produce a displacement of a whole wave is determined, and then that necessary to compensate for the extra fraction of a wave in each case.

Light is made incident just normal to the mirrors, thus producing fringes very sharply defined on one edge and appearing suddenly in the central spot. This is a large factor in the sensibility of the method. With care the un-

certainty in determination by this method need not exceed one in five to ten million.

Interference Refractometry.—The theory of interference refractometry was outlined in the chapter on Refractometry. Three of the best methods are here briefly described.

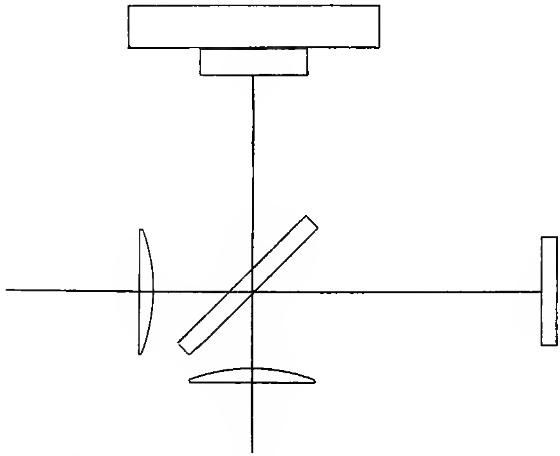

FIG. 71.—Michelson interferometer for observing slight displacements.

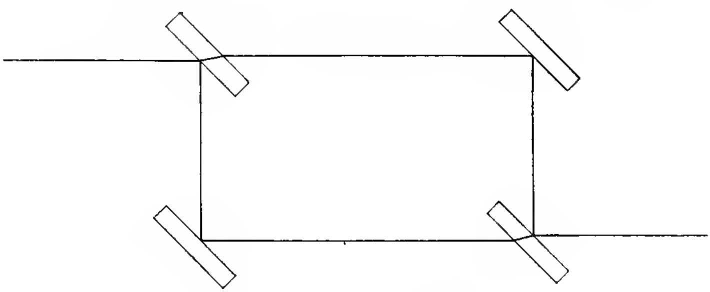

FIG. 72.—Jamin Interferometer.

The method in most general laboratory use for some time has been to simply insert a column of the refracting substance in one arm of a Michelson (Fig. 71) or Jamin refractometer. The Jamin arrangement has been given a variety of

forms, but the general scheme of dividing and recombining the beam is shown in the figure.

Some very sensitive and stable differential forms particularly adapted to gas refractometry have recently been developed by Löwe and outlined in the figure.

Rayleigh recently devised a very simple and minute laboratory refractometer; two holes 1/2 mm in diameter and 20 mm long cast side by side in paraffin contain the

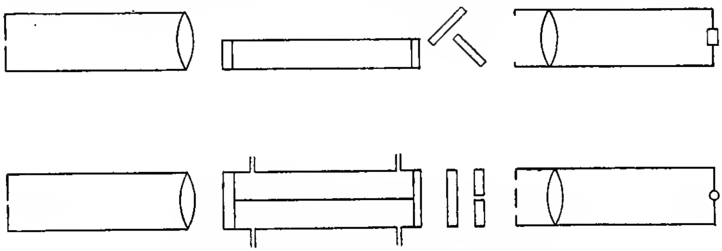

FIG. 73.—Diagram of Löwe Gas Refractometer.

gases of known and unknown index. These are placed longitudinally between collimator and telescope of an ordinary spectrometer. The precision attainable (in $n-1$) is better than 1 percent and the quantity of gas necessary is but a few cubic millimeters.

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